OPTIONS AND GUARANTEES IN LIFE INSURANCE AND PENSION PRODUCTS

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Agenda

1. Introduction
2. Packaging guarantees and options
3. Modelling issues
4. Building the post-retirement income
5. A range of annuity products
6. Some arrangements for the payout phase
7. The payment profile
8. Concluding remarks

1 INTRODUCTION

Nineteenth century: a large variety of policies, to some extent tailored on the personal needs of the insured, was customary in several European insurance markets

Later: a *standardization* process started $\Rightarrow$ a progressive shift to a very small set of standard products, basically:

- ▷ endowment insurance
- ▷ term insurance
- ▷ immediate life annuity
- ▷ deferred life annuity

Recently: to some extent, an inverse process is developing $\Rightarrow$ many modern insurance and pension products are designed as *packages*, whose items may be either included or not in the product actually purchased by the client

Interesting examples provided by:

- endowment insurance which can include various rider benefits and options
- Universal Life insurance
- Variable Annuities
- other insurance or financial products which eventually aim at constructing a post-retirement income
- presence of possible Long Term Care benefits in pension products (e.g. uplift of the annuity benefit)
- . . . . . .

Benefits provided imply a wide range of "guarantees" $\Rightarrow$ risks borne by the insurance company (or the pension fund)

Guarantees and inherent risks are clearly perceived in recent scenarios, in particular because of

- ▷ volatility in financial markets
- ▷ trends in mortality / longevity (and uncertainty in trends)

Appropriate modelling tools are then needed for pricing and reserving
$\Rightarrow$ logical and technical shift from expected present values, and their prominent role in life insurance and pension calculations, to more modern and complex approaches, like the ERM (Enterprise Risk Management) - based approach

Drawbacks (consequences of the shift):

- complexity is often an obstacle on the way towards sound pricing and reserving principles
- if sound pricing leads to very high premiums, the insurer's market share could become smaller

Alternative solution: appropriate *product design* aiming either

- at sharing risks between insurer and policyholders

or

- at transferring some risks to policyholders

An important example, as regards the market risk in participating (or with-profit) policies: shift from

▷ guarantee of ANNUAL MINIMUM INTEREST

to

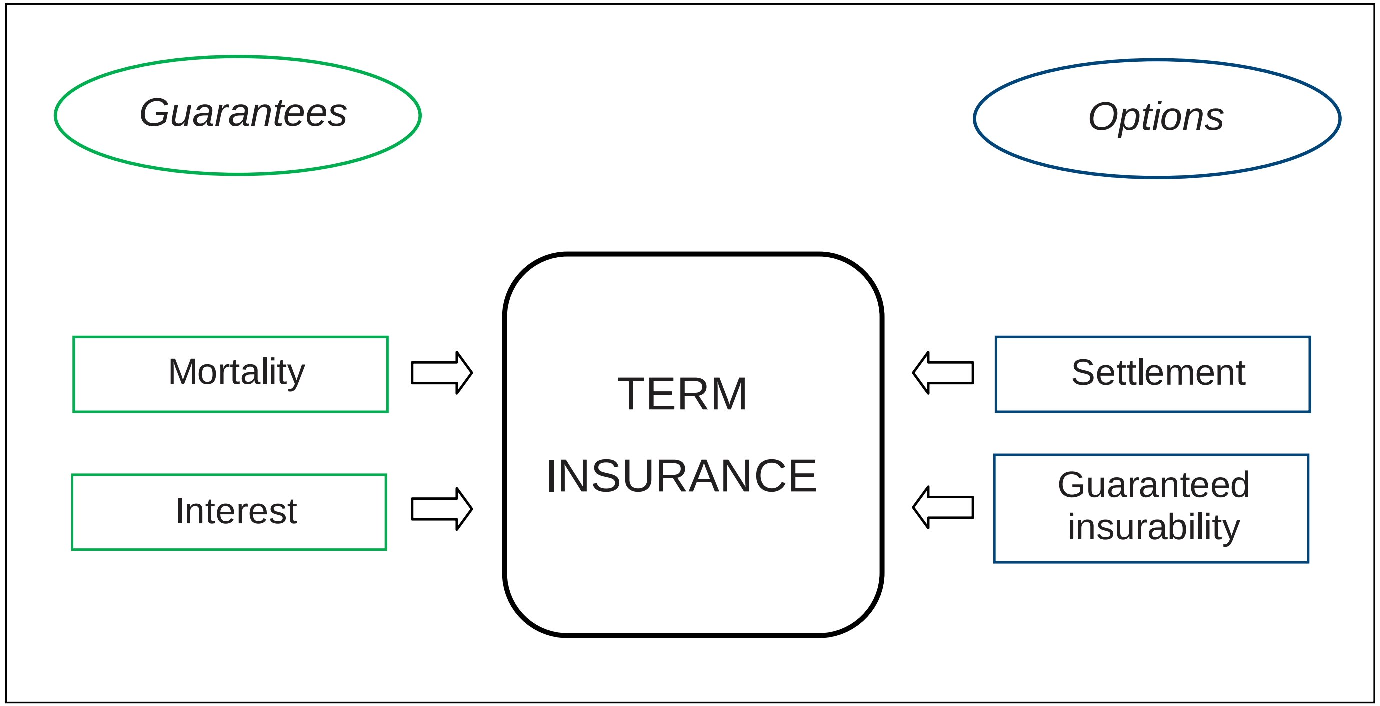
▷ guarantee of AVERAGE MINIMUM INTEREST (e.g. guarantee at maturity)

2 PACKAGING GUARANTEES AND OPTIONS

SOME EXAMPLES OF GUARANTEES AND OPTIONS IN INSURANCE AND PENSION PRODUCTS

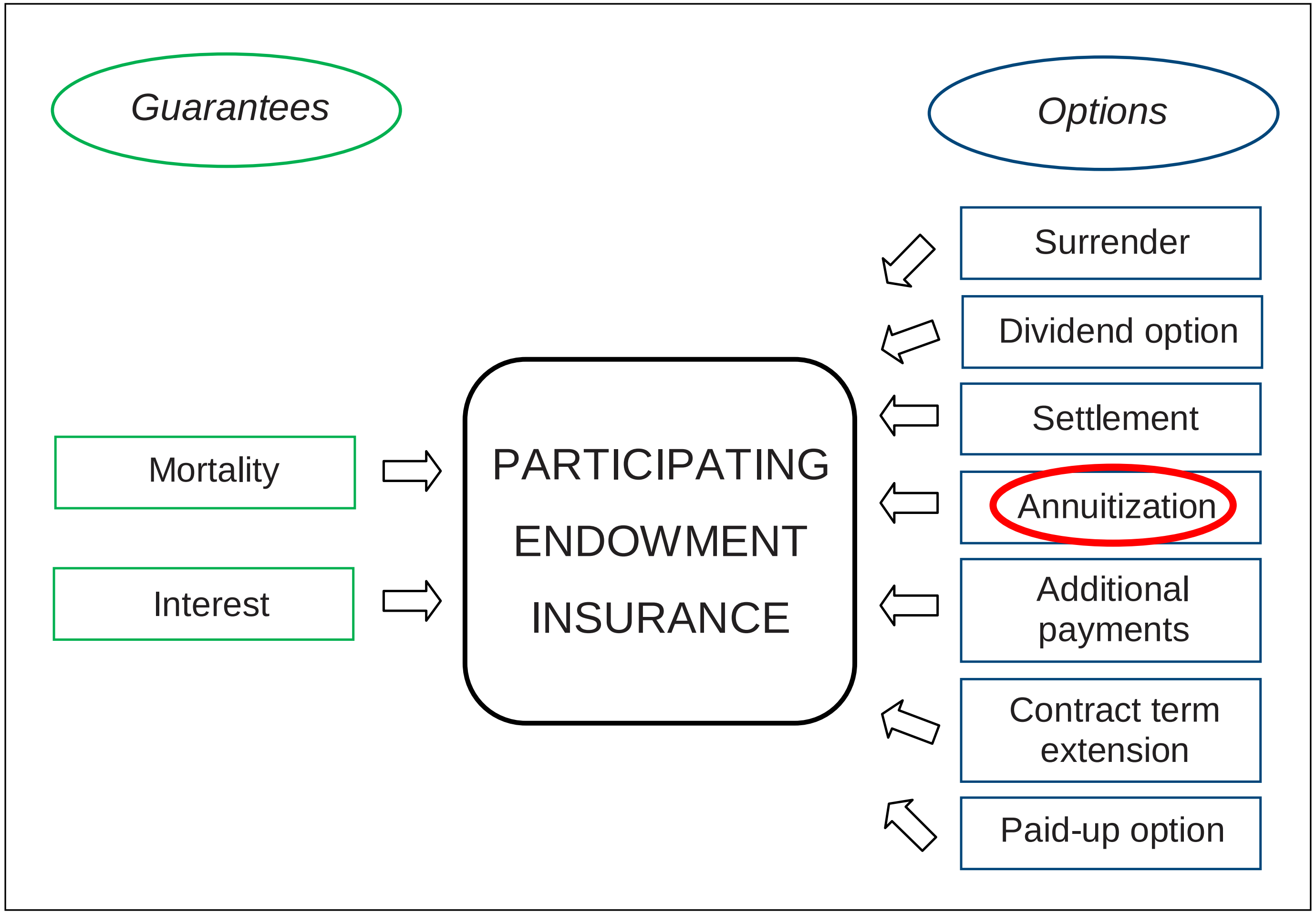
See, for example:

Black and Skipper [2000], Gatzert [2009], Hardy [2004]



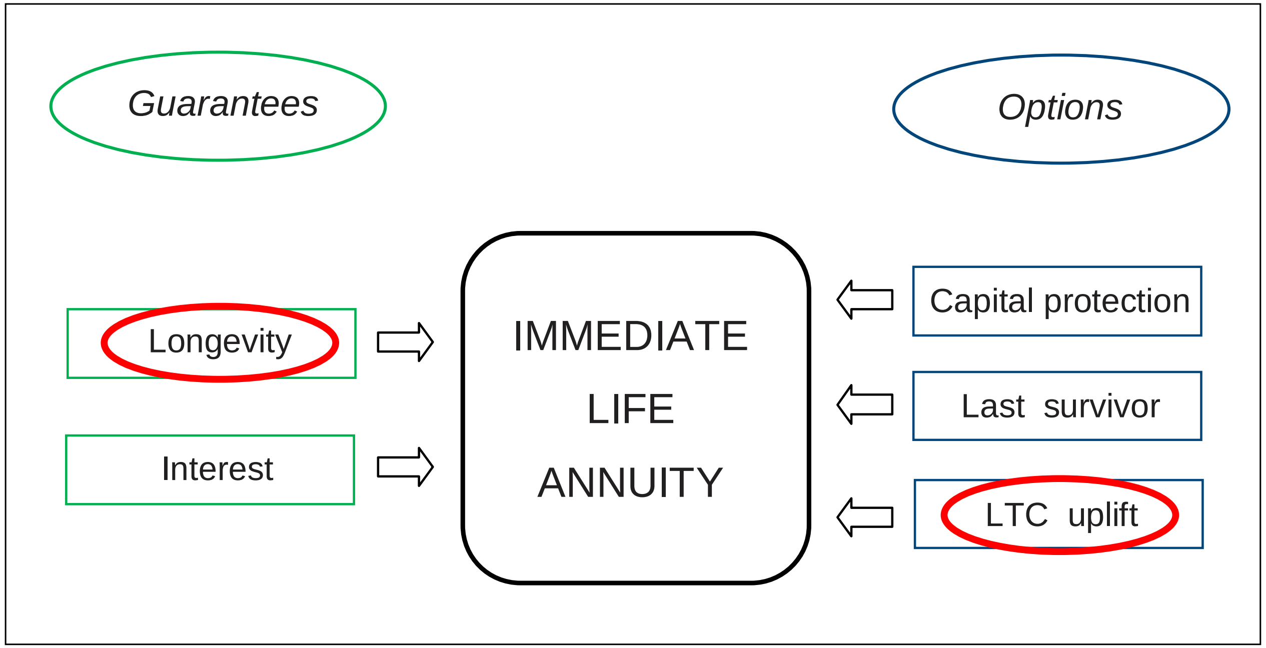
Examples of guarantees & options: the Term Insurance

Packaging guarantees and options *(cont'd)*



Examples of guarantees & options: the Endowment Insurance

Packaging guarantees and options *(cont'd)*

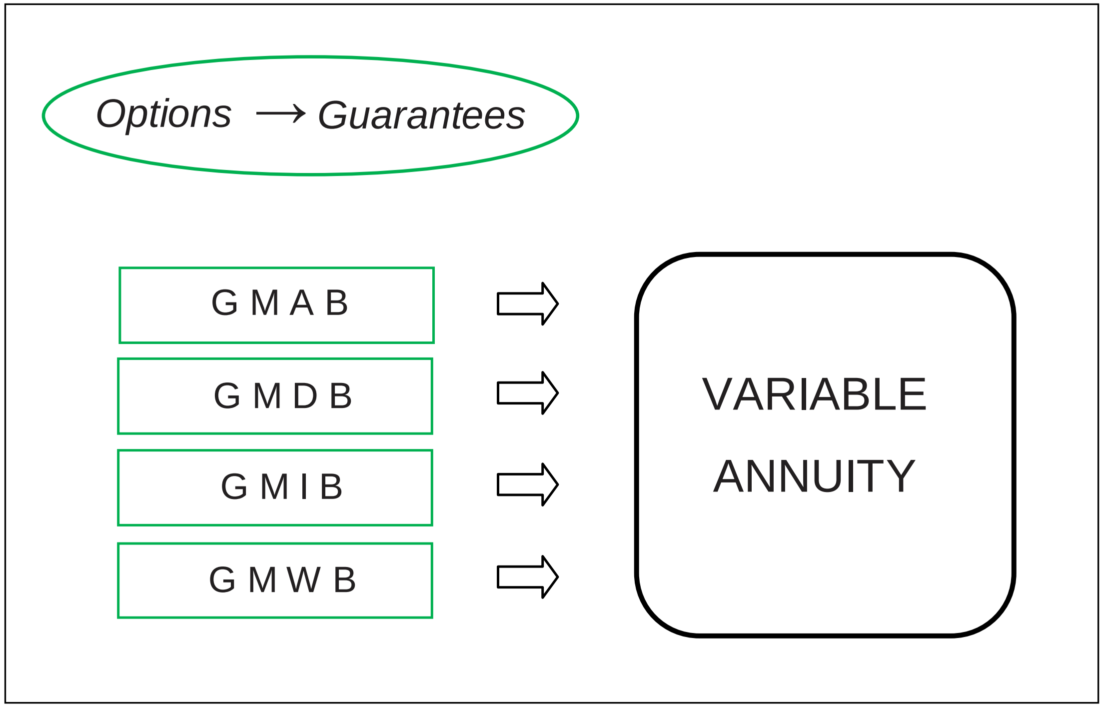


Examples of guarantees & options: the Immediate Life Annuity

In Variable Annuity products the presence of guarantees follows policyholder's choices

See, for example:

Bacinello et al. [2011], Kalberer and Ravindran [2009]



Examples of guarantees & options: the Variable Annuity

3 MODELLING ISSUES

THE EARLIEST FORMULAE

See, for example:

Haberman [1996], Hald [1987]

Actuarial value of an immediate life annuity

Edmond Halley (1693)

$$a_x = (1+i)^{-1}\,\frac{l_{x+1}}{l_x} + (1+i)^{-2}\,\frac{l_{x+2}}{l_x} + (1+i)^{-3}\,\frac{l_{x+3}}{l_x} + \dots$$

- computationally straightforward
- "modern" interpretation:

$$\frac{l_{x+h}}{l_x} = {}_hp_x = \mathbb{P}[T_x > h]$$

hence

$$a_x = (1+i)^{-1}\,{}_1p_x + (1+i)^{-2}\,{}_2p_x + (1+i)^{-3}\,{}_3p_x + \dots$$

Jan de Witt (1671)

$$a_x = a_{\overline{1}|} \frac{l_{x+1} - l_{x+2}}{l_x} + a_{\overline{2}|} \frac{l_{x+2} - l_{x+3}}{l_x} + a_{\overline{3}|} \frac{l_{x+3} - l_{x+4}}{l_x} + \dots$$

with

$$a_{\overline{h}|} = \frac{1 - (1+i)^{-h}}{i}$$

- "modern" interpretation

$$\frac{l_{x+h} - l_{x+h+1}}{l_x} = {}_h p_x \, q_{x+h} = \mathbb{P}[h < T_x \le h+1]$$

  hence

$$a_x = a_{\overline{1}|} \, {}_1 p_x \, q_{x+1} + a_{\overline{2}|} \, {}_2 p_x \, q_{x+2} + a_{\overline{3}|} \, {}_3 p_x \, q_{x+3} + \dots$$

- more interesting for further developments; can be read as

$$a_x = \mathbb{E}[a_{\overline{K_x}|}]$$

Actuarial value of an endowment insurance

$$A_{x,\overline{m|}} = (1+i)^{-1}\,\frac{l_x - l_{x+1}}{l_x} + (1+i)^{-2}\,\frac{l_{x+1} - l_{x+2}}{l_x} + \cdots + (1+i)^{-m}\,\frac{l_{x+m}}{l_x}$$
$$= (1+i)^{-1}\,q_x + (1+i)^{-2}\,{}_1p_x\,q_{x+1} + \cdots + (1+i)^{-m}\,{}_mp_x$$

Features of the underlying actuarial model

- Deterministic
  - ▷ although relying on probabilities, only *expected values of benefits* are finally addressed
  - ▷ possible impact of *risks originated by guarantees* (interest, mortality / longevity, etc) not (explicitly) accounted for
  - ▷ just implicit safety loading via adoption of prudential technical bases
- Static (implicitly)
  - ▷ the construction of the life table $l_x, l_{x+1}, \dots$ from observed mortality rates $\hat{q}_{x+h}$ relies on the assumption that the age pattern of mortality will not change in the future
- Single-decrement
- ... ...

Shift to more complex models required because of:

- ▷ awareness of the presence of *guarantees* which imply risks borne by the insurer
- ▷ the complexity of some products, also including *options*
- ▷ evolving scenarios (mortality / longevity, financial markets, inflation, etc.)
- ▷ the need for a sound assessment of the insurer’s risk profile

“Guidelines” can be suggested by the Enterprise Risk Management (ERM) approach (see Part 2 of this seminar)

In what follows, we only focus on singling-out the guarantees (in particular related to the longevity risk) implied by various life annuity arrangements

4 BUILDING THE POST-RETIREMENT INCOME

Introduction

We describe various arrangements, involving either the accumulation phase, or the payout phase, or both

Various products are available on financial and insurance markets, each product with a specific guarantee structure (conventional life annuities either immediate or deferred, Variable annuities, withdrawal plans, etc.)

See:

Shapiro [2010]

This research provides an extensive literature review of post-retirement financial strategies

See also:

Pitacco et al. [2009], Rocha et al. [2011], Wadsworth et al. [2001]

and references therein, for general issues on life annuities

Building the post-retirement income *(cont'd)*

We focus on guarantees provided by each arrangement

Risks taken by the intermediary, in particular the annuity provider (either insurer or pension fund) immediately identified looking at the guarantee structure

In the following figures:

- x = age at policy issue, or at entering the pension scheme
- $x + r$ = age at retirement

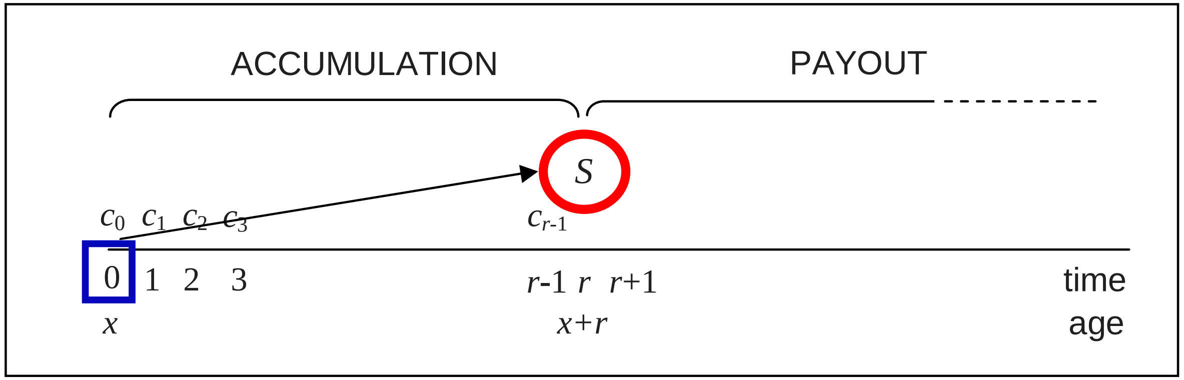
□ Time at which the guarantee is stated

○ Ultimate object of the guarantee

Some basic structures

Structure 1 - Accumulation phase only

Given the sequence of contributions / premiums / savings
$c_0, c_1, \ldots, c_{r-1} \;\Rightarrow$ amount S guaranteed

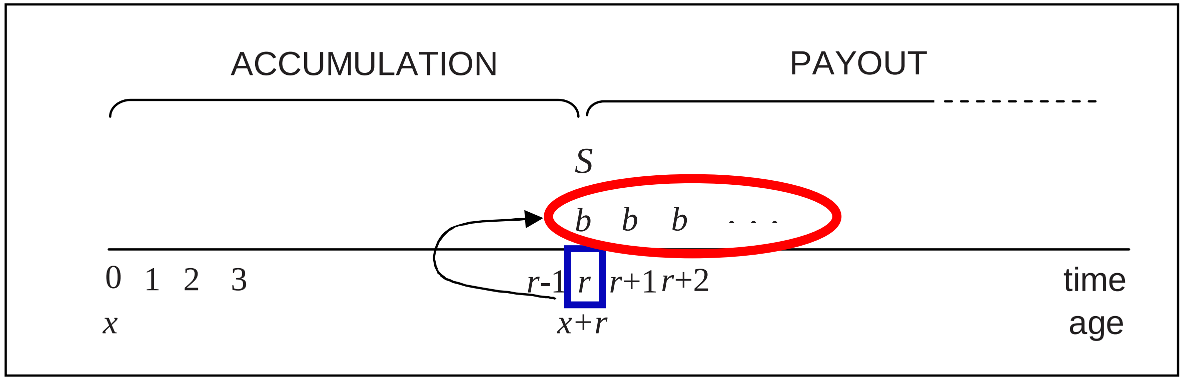


Examples

- financial product: interest guarantee
- insurance product, e.g. pure endowment insurance or endowment insurance: interest guarantee and mortality guarantee

Structure 2 - Payout phase only

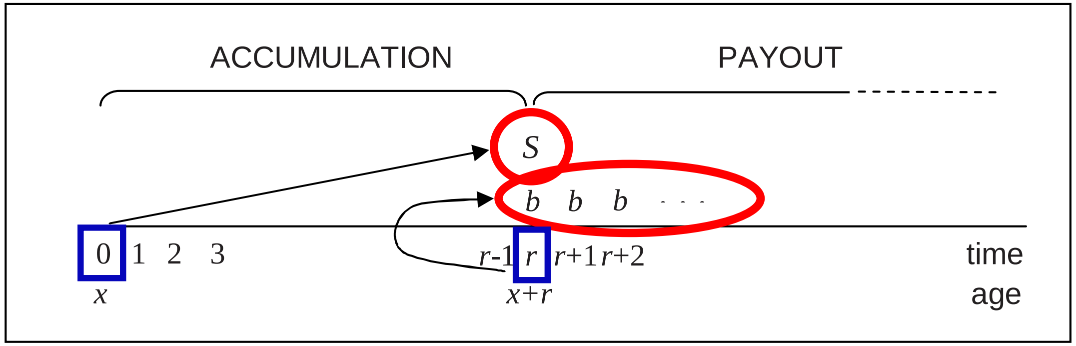
Given the amount S $\Rightarrow$ annual benefit b guaranteed (assuming a flat payment profile)



Examples

- financial product: interest guarantee $\Rightarrow$ annual benefit b guaranteed up to fund exhaustion (at a defined time)
- insurance product, i.e. a CAR immediate life annuity: interest guar. and mortality guar. $\Rightarrow$ benefit b guaranteed lifelong $\Rightarrow$ *longevity guarantee* (CAR = current annuity rate, i.e. at time r)

Structure 3 - Accumulation phase + Payout phase (combining structure 1 and 2)

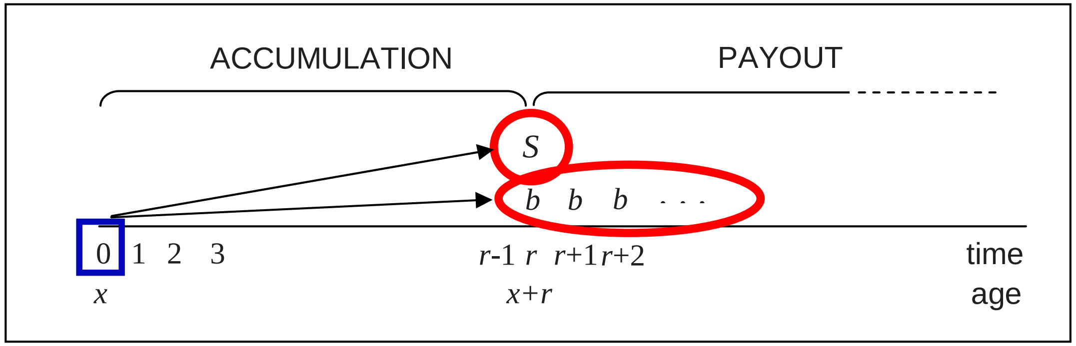


Examples

- financial product for the accumulation phase: interest guarantee $\Rightarrow$ S guaranteed
- insurance product, i.e. a CAR immediate life annuity for the payout phase: for any given S, interest guarantee and mortality guarantee $\Rightarrow$ benefit b guaranteed lifelong

Structure 4 - Accumulation phase + Payout phase

All guarantees stated at time 0 (a challenge for the annuity provider !)



Examples

- GAR deferred life annuity (GAR = guaranteed annuity rate. i.e. at time 0)

  Remark

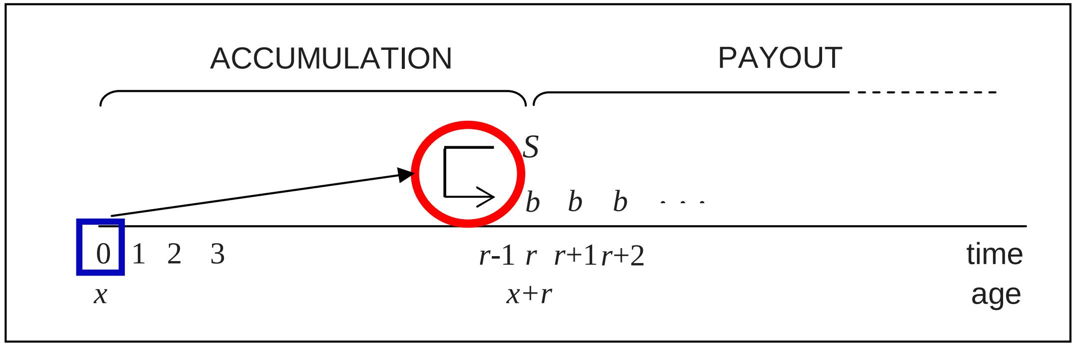
  Structure implied in particular by the classical actuarial formula

  $P\,\ddot{a}_{x:\overline{r}|} = b\;{}_{r|}\ddot{a}_x \;\Rightarrow\; S = \ddot{a}_{x+r} =$ policy reserve at time r

- financial product with interest guarantee for the accumulation phase and GAR immediate life annuity for the payout phase

Structure 5 - Accumulation phase + Payout phase

Conversion rate stated at time 0



Example

- financial product for the accumulation phase and immediate life annuity for the payout phase; guaranteed conversion rate

  Remark

  In particular: GAO product, providing the options (at retirement):
  - ▷ lump sum
  - ▷ annuitization at CAR
  - ▷ annuitization at GAR

Remark 1

Assume that the accumulation phase works according to the logic of single recurrent premiums (that is, a particular progressive funding of S)

Then, guarantees in both Structure 4 and Structure 5 can be weakened by linking the guarantee specification (the accumulation guarantee and/or the conversion rate) to each single recurrent premium

Remark 2

Starting from the basic structures (see above) it is possible to conceive product design by moving in various directions; in particular:

- ▷ reducing the "scope" of some guarantees, viz the longevity guarantee
- ▷ designing a non-guaranteed product, allowing for the inclusion of one or more guarantees, chosen by the client $\Rightarrow$ Variable Annuities and GMxB

See what follows

5 A RANGE OF ANNUITY PRODUCTS

We describe two specific products:

- ▷ Advanced Life Delayed Annuity (ALDA)
- ▷ Ruin Contingent Life Annuity (RCLA)

and one “category” of products:

- ▷ Variable Annuities

All these products involve both accumulation phase and payout phase

Advanced Life Delayed Annuity (ALDA)

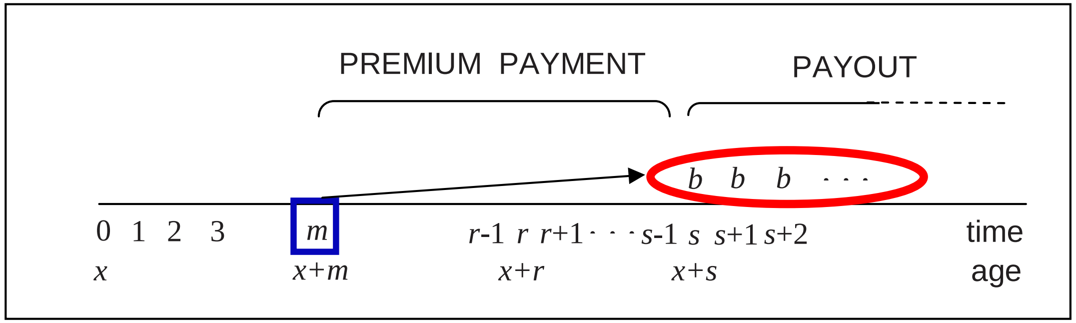
The premium payment period does not necessarily coincide with the (traditional) accumulation phase, being possibly shifted towards older ages

The payout period starts after retirement time (age 80 or 85, say)

$\Rightarrow$ withdrawal from a fund throughout the time interval $(r, s-1)$ to get post-retirement income

See:

Milevsky [2005], Gong and Webb [2010], Stephenson [1978]



See Structure 4, adapted by shifting: $0 \rightarrow m$, $r \rightarrow s$

Purposes of ALDA:

- to provide longevity insurance at old ages only (that is, insurance cover with a deductible)
- to pay (possibly) an inflation-adjusted income
- to reduce premium amount (with respect to conventional deferred annuities)
- to enhance rates of voluntary annuitization, thanks to lower premiums

Ruin Contingent Life Annuity (RCLA)

The post-retirement income is provided by

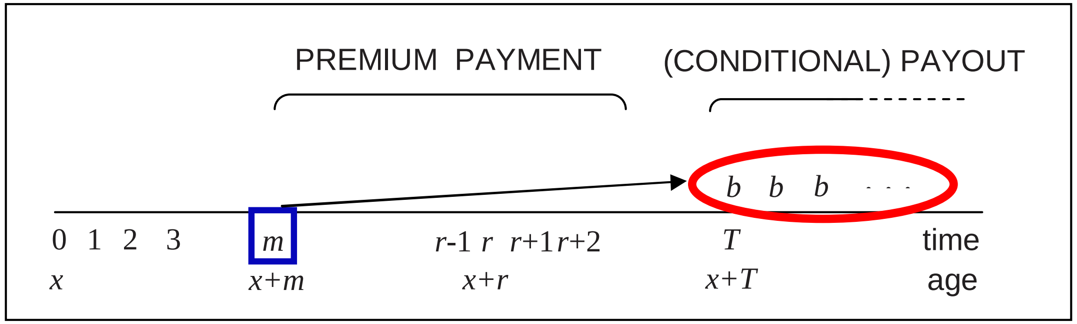
(1) withdrawal from a fund from time r onwards, up to (possible) exhaustion of the fund

(2) a life annuity paid to the retiree from (random) time T of fund exhaustion because of "adverse" scenario

- ▷ poor performance of the fund
- ▷ long lifetime

See:

Huang et al. [2009]



RCLA can be thought as

(a) an ALDA with random delay $T - r$, and trigger given by the scenario

(b) an insurance product generating annuitization as a worst case scenario

Pricing RCLA $\Rightarrow$ need for constructing a pseudo-index, accounting for

- the behaviour of a market performance index
- a set of reasonable withdrawal rates throughout the payout phase

Variable Annuities (VA)

An investment product (throughout the accumulation phase), then providing a post-retirement income

No guarantee is implicitly embedded

Various guarantees (GMxB = Guaranteed Minimum Benefit of type x) can be choosen by the client and then included

See, for example:

Bacinello et al. [2011], Kalberer and Ravindran [2009], Pitacco [2012]

and references therein

Including guarantees logically results in structures we have defined above

In what follows we disregard the Guaranteed Minimum Death Benefit (GMDB)

Let F_t denote the balance (fund value) at time t

Guaranteed Minimum Accumulation Benefit (GMAB)
(referring for simplicity to a single premium $\mathit{\Pi}$)

- *return of premiums* $G_r^{[\mathrm{A}]} = \mathit{\Pi}$
- *roll-up guarantee* $G_r^{[\mathrm{A}]} = \mathit{\Pi}\,(1+i')^r$
- *ratchet guarantee* $G_r^{[\mathrm{A}]} = \max_{t_h<r}\{F_{t_h}\}$
  where t_h, $h = 1, 2, \dots$ are stated times
- *reset guarantee* $G_r^{[\mathrm{A}]} = F_{\max\{t_j:\, t_j<r\}}$
  where t_j, $i = 1, 2, \dots$ are the stated reset times

See Structures 1, 3 and 4:

$$S \geq G_r^{[\mathrm{A}]}$$

Guaranteed Minimum Income Benefit (GMIB)

Provides a life annuity, i.e. a lifelong post-retirement income

Two possible arrangements

(1) Amount to annuitize; then

$$b^{[\mathrm{I}]} = \frac{1}{\ddot{a}_{x+r}^{[\mathrm{CAR}]}} \max\{F_r, G_r^{[\mathrm{I}]}\}$$

where $G_r^{[\mathrm{I}]}$ can be defined as $G_r^{[\mathrm{A}]}$

See Structure 3:

$$S \geq G_r^{[\mathrm{I}]}$$

(2) Annuitization rate; then

$$b^{[\mathrm{I}]} = F_r \ \max\left\{ \frac{1}{\ddot{a}_{x+r}^{[\mathrm{CAR}]}}, \frac{1}{\ddot{a}_{x+r}^{[\mathrm{GAR}]}} \right\}$$

Guarantee aka GAO

See Structure 5

In principle, the two guarantees can be combined; in practice, resulting product very expensive, because of insurer's huge risk

(3) Amount & annuitization rate; then

$$b^{[\mathrm{I}]} = \max\{F_r, G_r^{[\mathrm{I}]}\} \ \max\left\{ \frac{1}{\ddot{a}_{x+r}^{[\mathrm{CAR}]}}, \frac{1}{\ddot{a}_{x+r}^{[\mathrm{GAR}]}} \right\}$$

See Structure 4

Guaranteed Minimum Withdrawal Benefit (GMWB)

Guaranteed benefits even in the case of fund exhaustion because of

- ▷ poor investment performance
- ▷ long lifetime

The guarantee affects both

- benefit amount
- benefit duration
  - (i) fixed
  - (ii) fixed provided that the retiree is alive
  - (iii) lifelong

Guaranteed duration (iii) $\Rightarrow$ logical structure of RCLA

6 SOME ARRANGEMENTS FOR THE PAYOUT PHASE

Basic features of the life annuity product

1. The life annuity relies on the mutuality mechanism; hence:
   (a) amounts released by the deceased annuitants are shared among the annuitants still alive $\Rightarrow$ mortality credits
   (b) on the annuitant's death, her / his estate not credited with any amount (no bequest available)
2. A life annuity provides an "inflexible" income (annual amounts cashed by the annuitant must be in line with the payment profile, as stated by policy conditions, or by pension plan rules)

Features 1(b) and 2: possibly perceived as disadvantages $\Rightarrow$ weaken propensity to immediately annuitize the whole amount available at retirement.

Disadvantages can be mitigated:

- ▷ purchasing a particular product (life annuity + other benefits)
- ▷ adopting a specific annuitization strategy

Life Annuity with a Guarantee Period

Temporary annuity-certain (throughout the guarantee period)
+ deferred life annuity

	Guarantee period		
	0	5	10
$x + r = 65$	$18\,070$	$18\,131$	$18\,386$
$x + r = 70$	$15\,265$	$15\,376$	$15\,832$

Single premium at retirement age; $b = 1\,000$

Value-Protected Life Annuity (i.e. with "capital protection")

In case of early death of the annuitant $\Rightarrow$ difference (if positive) between single premium and cumulated benefits paid to the annuitant is paid to the beneficiary

Usually, capital protection expires at some given limit age

	Limit age		
	70	75	80
$x + r = 65$	$18\,596$	$19\,213$	$19\,807$
$x + r = 70$	$15\,265$	$16\,062$	$16\,936$

Single premium at retirement age; $b = 1\,000$

Some arrangements for the payout phase *(cont'd)*

Remark

In both the products *Life Annuity with a Guarantee Period* and *Value-Protected Life Annuity* the extra-premium is small or very small, depending on the extension of the rider benefit

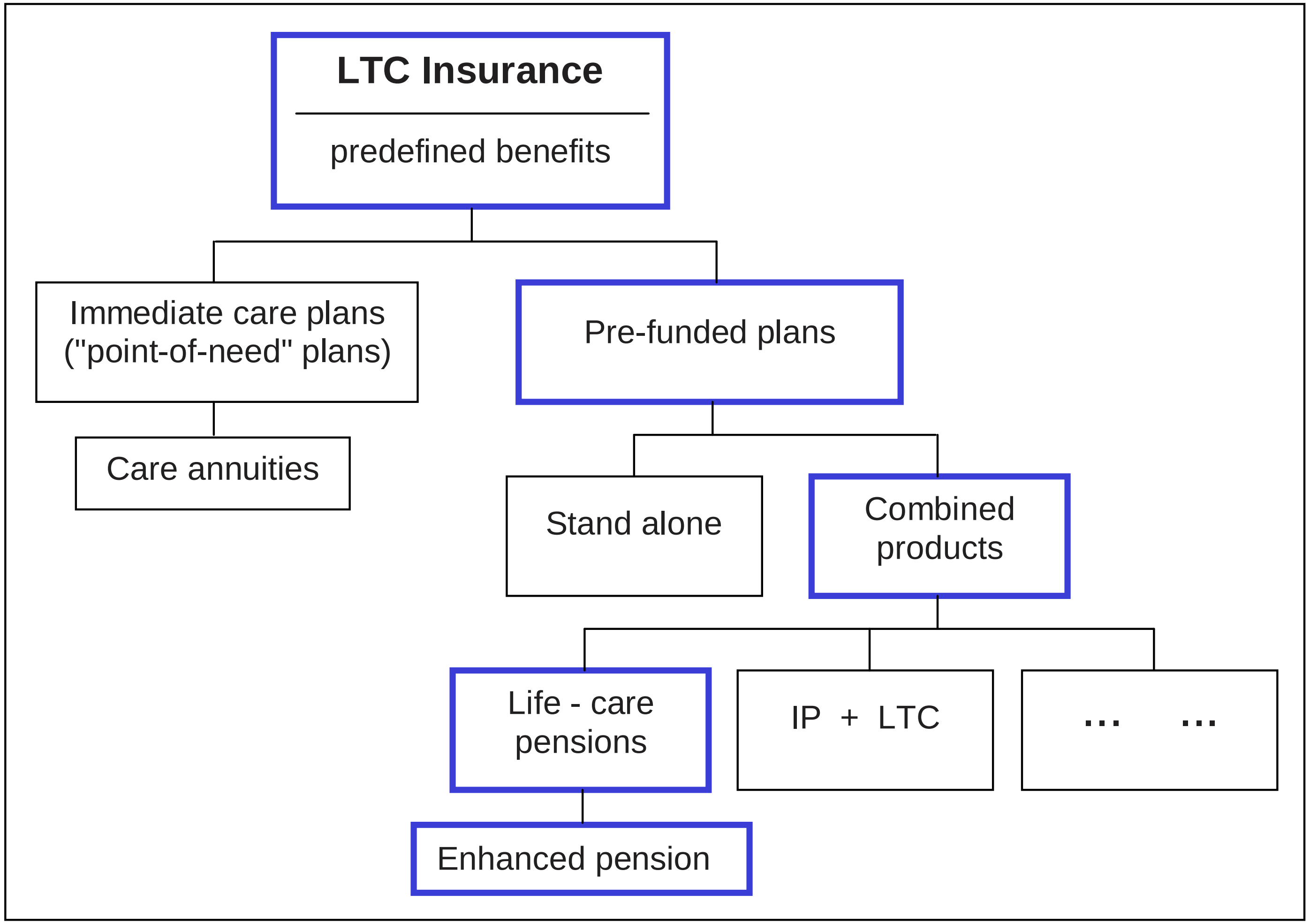
Obvious reason: the mortality in the age intervals involved is small or very small

Under the annuity provider's perspective: capital protection (i.e. a death benefit) does not provide an effective hedge against the (aggregate) longevity risk

Natural hedging of the aggregate longevity risk (both *across LOBS* and *across time* as well) remains a difficult issue !

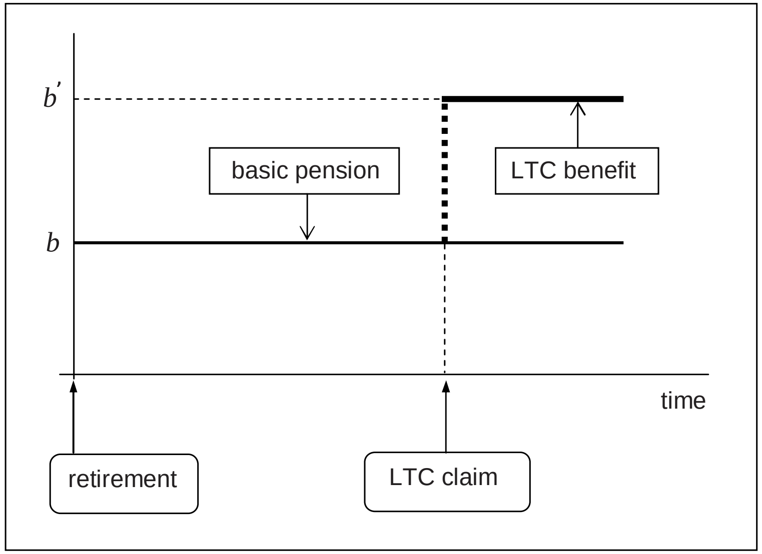
Some arrangements for the payout phase *(cont'd)*

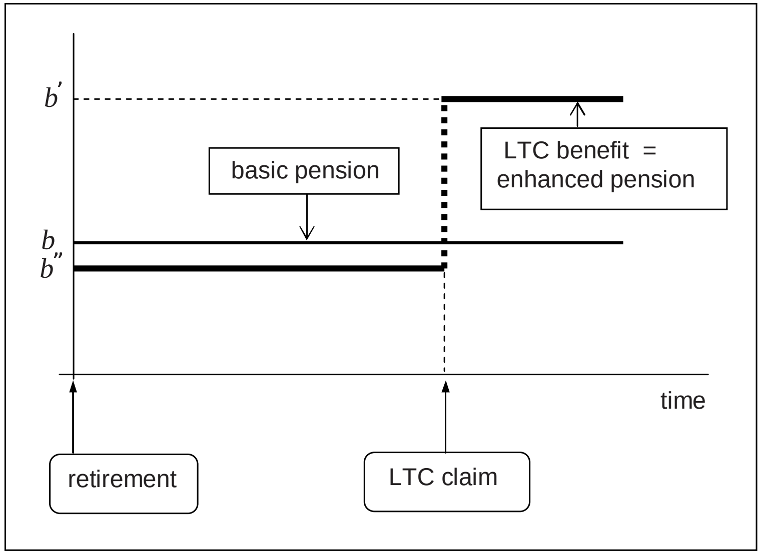
Annuity products providing LTC benefits



Some arrangements for the payout phase *(cont'd)*

The Life Care Annuity





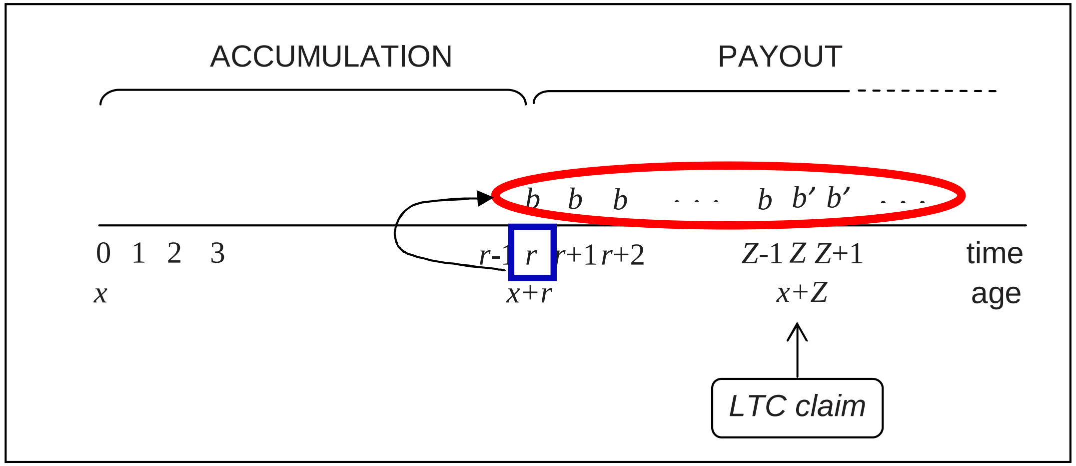
The Enhanced Pension:
Life Care Annuity financed via reduction of the basic pension

Life Care Annuity

A health-related product: in the case of Long Term Care need $\Rightarrow$ shift from the basic benefit b to b' ($b' > b$)

See, for example:
Warshawsky [2007], Zhou-Richter and Gründl [2011]
and references therein

Life annuity with benefit $b' - b$ $\Rightarrow$ logical structure of RCLA conditional on health status (but different financial structure !)



Purpose: to reduce the prevailing risk feature of the stand-alone LTC annuity

A further option: financing the uplift via reduction of the basic benefit
$\Rightarrow$ *Enhanced pension*

See, for example:

Haberman and Pitacco [1999]

and references therein

Packaging LTC annuity and ALDA

Insurance package including:

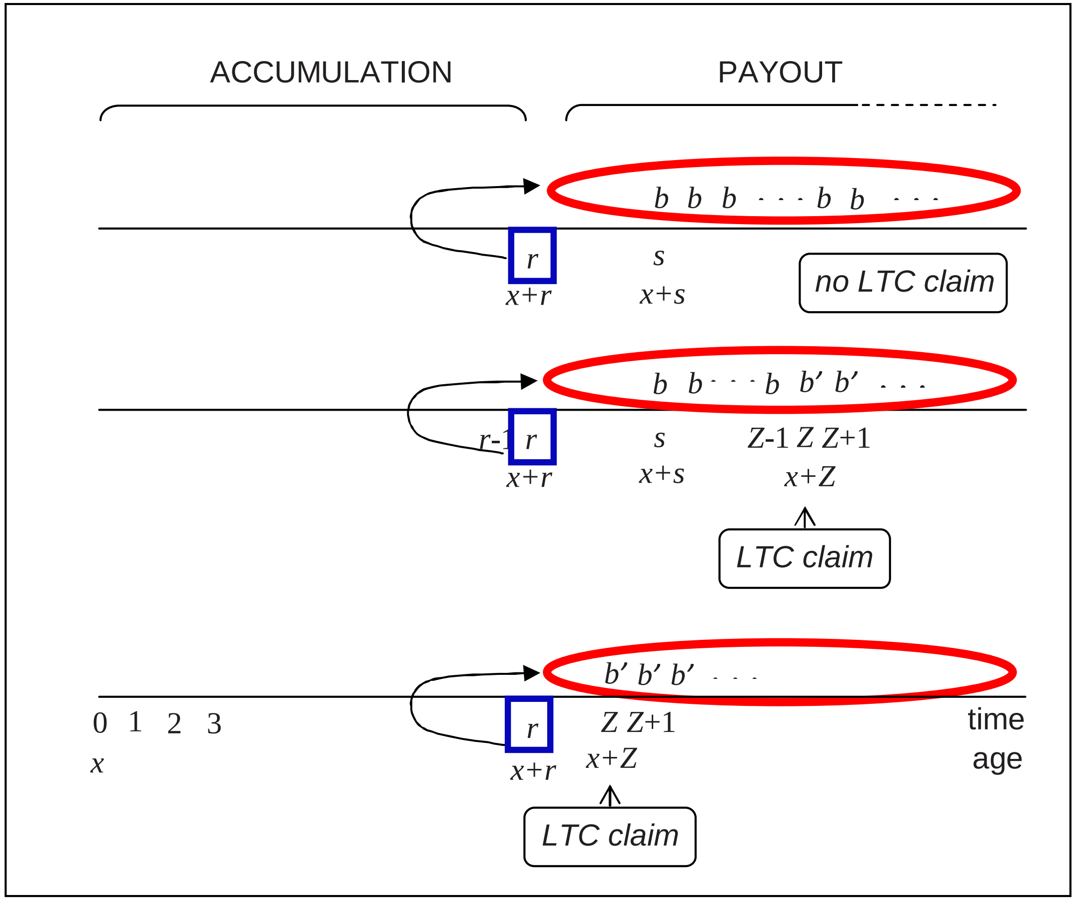
(1) LTC annuity

(2) deferred life annuity (e.g. from age 80), while the insured is not claiming LTC benefits

See following Figure

Another example of product design aiming at a reduction of the prevailing risk feature of the stand-alone LTC annuity

Some arrangements for the payout phase *(cont'd)*



Possible outcomes, depending on lifetime and LTC need

Some arrangements for the payout phase *(cont'd)*

Remark

When a Life Care annuity or a LTC annuity is involved, a specific type of aggregate longevity risk is taken by the annuity provider, inherent the lifetimes of elderly people claiming for LTC

Various theories concerning the relation between trend in expected total lifetime and trend in expected healthy lifetime

See, for example:
Olivieri and Ferri [2003]
and references therein

Progressive annuitization

See:
Blake and Hudson [2000], Blake et al. [2003], Horneff et al. [2008],
Milevsky and Young [2002]

Assume that, at time of retirement, amount S available to the retiree

The retiree can choose between two alternatives:

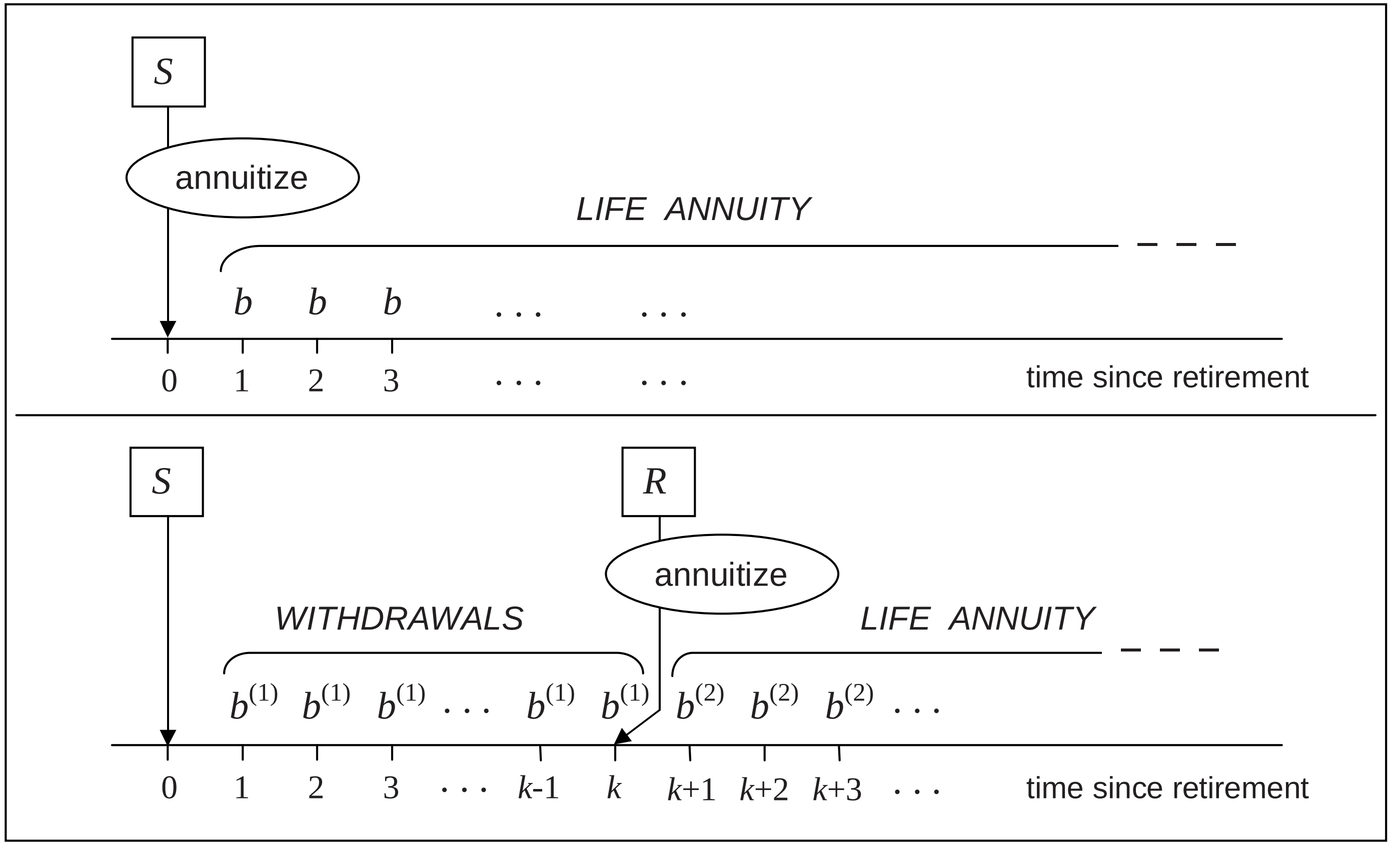
(1) to purchase an immediate life annuity, with annual benefit b (i.e. to annuitize amount S); see Figure, upper panel

(2) to leave amount S in a fund, and then

   (a) withdraw the amount $b^{(1)}$ at times $h = 1, 2, \dots, k$ (say, with $k = 5$ or $k = 10$) $\Rightarrow$ temporary withdrawal process

   (b) convert at time k the remaining amount R into an immediate life annuity with annual benefit $b^{(2)}$ $\Rightarrow$ delayed annuitization (provided she / he is alive); see Figure, lower panel

Some arrangements for the payout phase *(cont'd)*



Immediate annuitization versus delayed annuitization

Advantages of delay in the purchase of the life annuity:

- in the case of death before time k, the fund available constitutes a bequest
- more flexibility gained, as the annuitant may change her / his income profile modifying the withdrawal sequence (however, with possible change in the fund available at time k)

Disadvantages:

- a higher interest rate than that provided by the annuity, to recover the absence of mortality credits (i.e. absence of mutuality)
- risk of a shift to a different life table in the pricing basis $\Rightarrow$ conversion rate at time k possibly less favorable to the annuitant
- if k is high, difficult to gain the required yield avoiding too risky investments

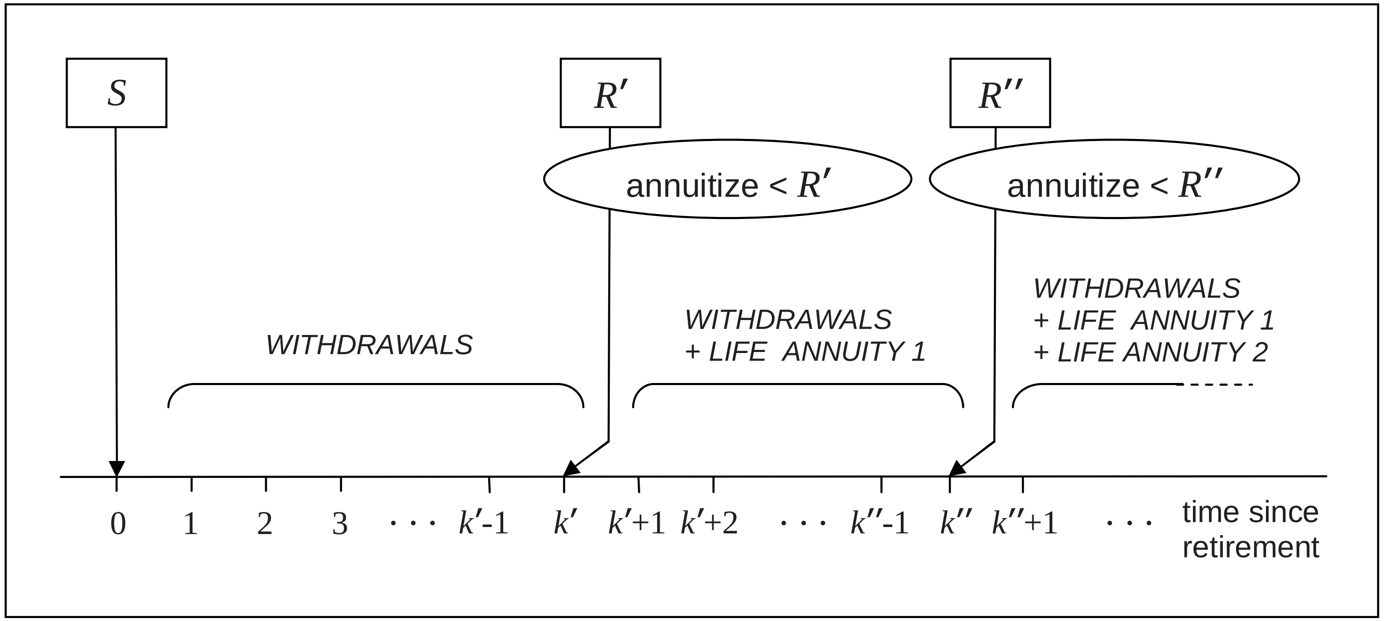
Interest rate $g(k)$ needed to recover mortality credits lost in $(0, k)$ ($i = 0.02$)

k	$g(k)$
5	0.02748
10	0.03009
15	0.03336
20	0.03718

Interest rate $g(k) \Leftrightarrow b^{(1)} = b^{(2)} = b$

Delayed annuitization $\Rightarrow$ trade-off between mortality risk and financial risk (and longevity risk as well, because of possible change in the annuitization rate)

A more general arrangement:



Staggered annuitization

7 THE PAYMENT PROFILE

So far we have focussed on *level annuities* $\Rightarrow$ income which is constant in nominal terms

A number of models of “varying” annuities have been derived, mainly with the purpose of protecting the annuitant against the loss of purchasing power because of inflation

In particular:

1. *Fixed-rate escalating annuities* (or *constant-growth annuities*)
2. *Index-linked annuities*
   - 2.a Inflation-linked annuities
   - 2.b Equity-indexed annuities
3. *Investment-linked annuities*
   - 3.a With-profit annuities (UK)
   - 3.b Annuities with profit participation mechanisms
   - 3.c Unit-linked annuities

The payment profile *(cont'd)*

Participation mechanisms (3.b) can involve both financial and mortality experience

Possible problem: poor mortality experience because of unexpected increase in longevity $\Rightarrow$ aggregate longevity risk

In traditional life annuity and pension design, the longevity risk is borne by the annuity provider

Alternative product design $\Rightarrow$ transfer part of the longevity risk to the annuitants $\Rightarrow$ definition of a *longevity-linked life annuity*

Sharing the (aggregate) longevity risk

Formally: *Adjustment process* $\Rightarrow$ benefit b_t due at time t:

$$b_t = b_0 \, \alpha_t^{[\mathrm{m}]}$$

with $\alpha_t^{[\mathrm{m}]}$ = coefficient of adjustment over $(0, t)$, according to mortality trend measure $[\mathrm{m}]$

Coefficient $\alpha_t^{[\mathrm{m}]}$ can incorporate investment profit participation
$\Rightarrow$ longevity loss can be offset by investment profit

Various interesting contributions regarding practicable models for the adjustment process and the measure $[\mathrm{m}]$

See:

Denuit et al. [2011], Goldsticker [2007], Kartashov et al. [1996], Lüty et al. [2001], Olivieri [2013], Piggott et al. [2005], Richter and Weber [2011], Rocha et al. [2011], Sherris and Qiao [2011], van de Ven and Weale [2008], Wadsworth et al. [2001]

8 CONCLUDING REMARKS

Actuarial mathematics and technique traditionally focussed on "benefits" in terms of the relevant expected present value ($\Rightarrow$ basically, a deterministic approach)

Risks implied by *guarantees* and *options* provided by policy conditions and pension plan rules usually disregarded (or, at least, not explicitly accounted for)

Current scenarios (market volatility and uncertainty in longevity dynamics) $\Rightarrow$ careful consideration of risks inherent in the life annuity and pension structures

Purpose of this presentation: to focus (according to ERM guidelines) on *risk identification* and *product design* looking at possible risk (in particular, biometric risk) transfers between annuitants and annuity provider

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