RISK MANAGEMENT FOR LIFE ANNUITIES IN A LONGEVITY RISK SCENARIO

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Agenda

1. Motivation
2. Cash flows in a life annuity portfolio
3. Uncertainty in future mortality trends
4. The ERM framework
5. Product design: sharing the longevity risk between annuitants and annuity providers
6. Concluding remarks

Presentation based on research and teaching material,
mainly joint with Annamaria Olivieri (University of Parma)

1 MOTIVATION

Just to provoke . . .

Are we sure that we know, for example:

what is the meaning of a “life table” ?

how to use life tables ?

what are the risks inherent in a life annuity ?

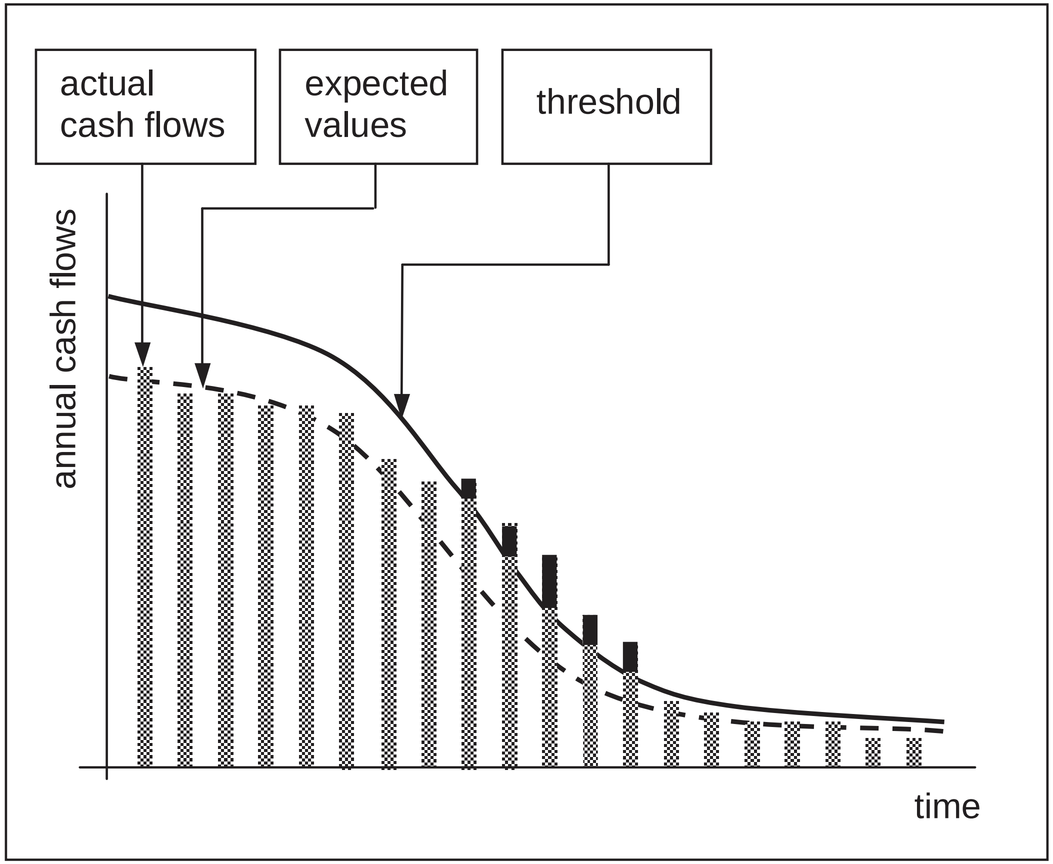
how to share longevity risk ?

. . . . . .

I’ll try to provide appropriate answers, looking at technical problems of life annuities under a (quantitative) risk-management perspective, implying the adoption of a stochastic approach to actuarial calculations

2 CASH FLOWS IN A LIFE ANNUITY PORTFOLIO

Refer to a cohort of immediate life annuities (single premium)



Annual cash flows in an annuity portfolio

(1) Why fluctuations / deviations of actual cash flows, if compared to expected cash flows ?

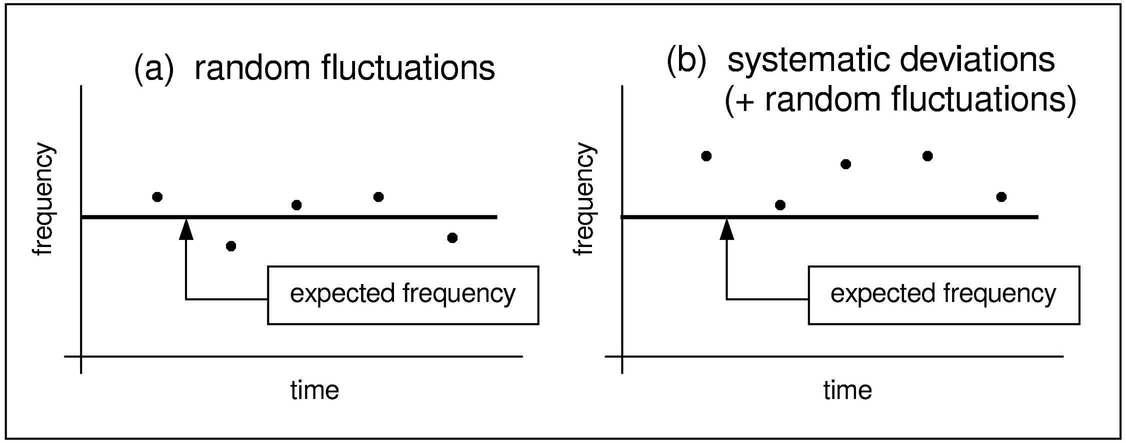
(2) What actions are available in order to
   - ▷ raise the threshold
   - ▷ modify the cash flow profile (reduce, smooth, etc.) ?

To provide answers:

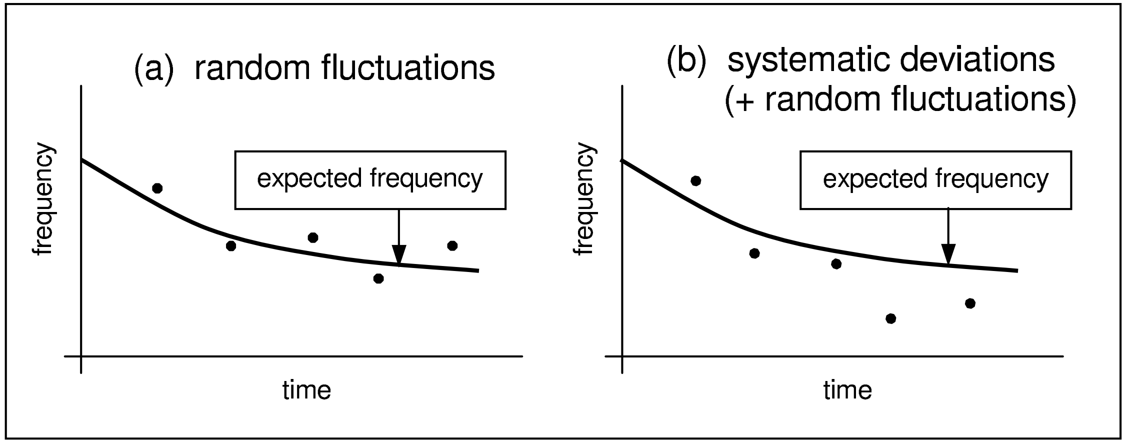
(1) look at mortality dynamics, and single-out risk components

(2) look at Risk Management actions, aiming at "risk mitigation"

Cash flows in a life annuity portfolio *(cont'd)*



Stationarity assumption



Trend assumption

3 UNCERTAINTY IN FUTURE MORTALITY TRENDS

THE AGE PATTERN OF MORTALITY

Traditional life tables and survival functions: the ultimate results of a statistical process starting from mortality observations

$$l_{x+1} = l_x \, (1 - q_x) \quad \text{for} \;\; x = 0, 1, \dots$$

$$S(x) = e^{-\int_0^x \mu_t \, dt} \quad \text{for} \;\; x > 0$$

(in a time-discrete and time-continuous context respectively)

If q_x, μ_x are based (as usual) on period observation
$\Rightarrow$ l_x, $S(x)$ rely on "static" mortality assumption

Sequences of (period) statistical observations witness mortality dynamics

Uncertainty in future mortality trends *(cont'd)*

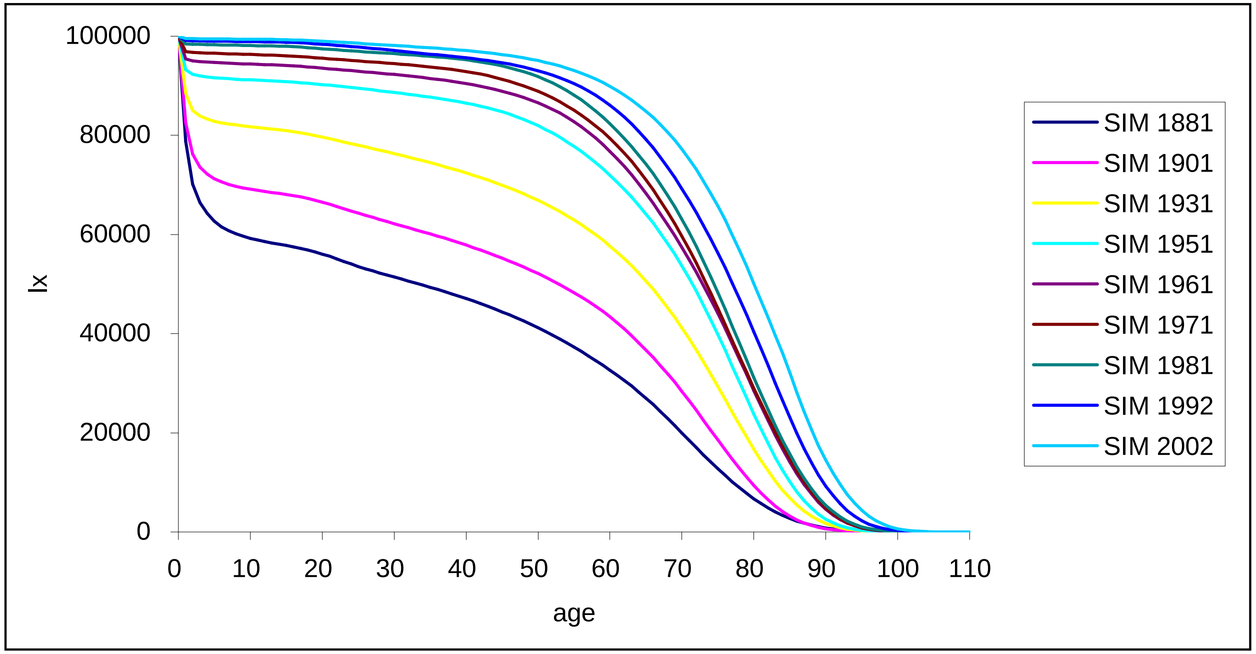
Given mortality dynamics, if we only rely on a period observation:

- what about the meaning of $\overset{\circ}{e}_0$?
- what about the meaning of a_{65} ?
- actuarial calculations should be restricted to "short" intervals

Projected life tables are needed to extend time intervals referred to, in particular for life annuities and other lifelong benefits (e.g. lifelong sickness covers, LTC benefits, etc.)

Do projected life tables fulfill all the requirements arising from current longevity scenario ?

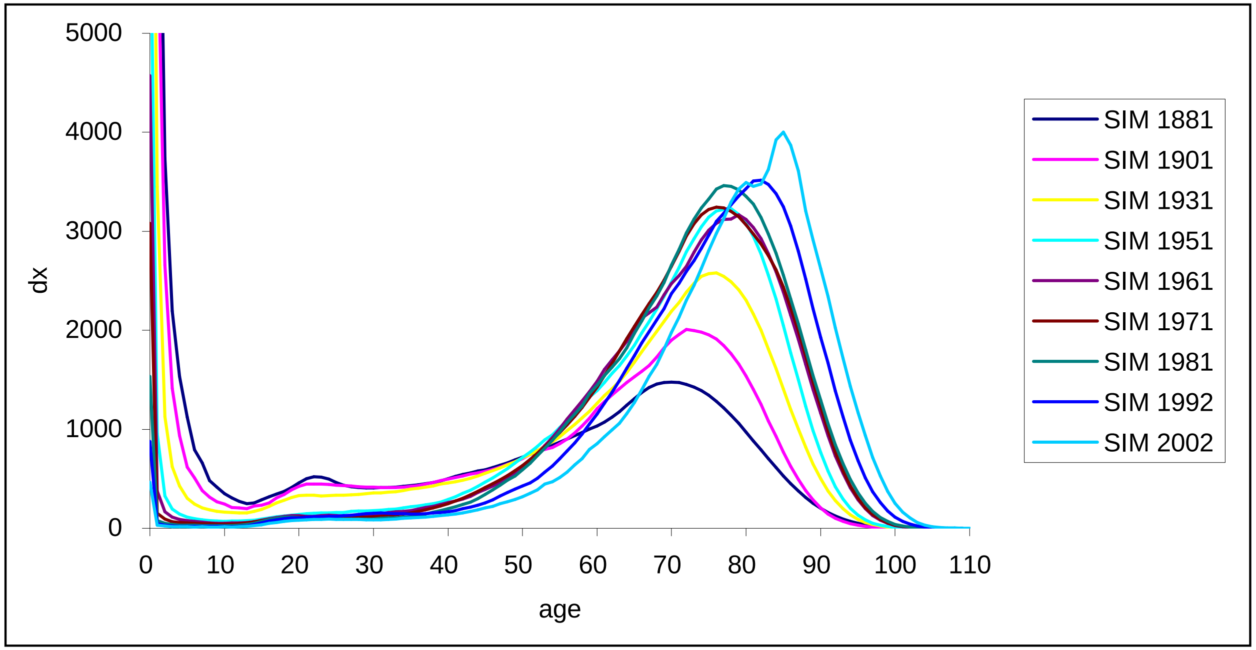
MORTALITY ON THE MOVE



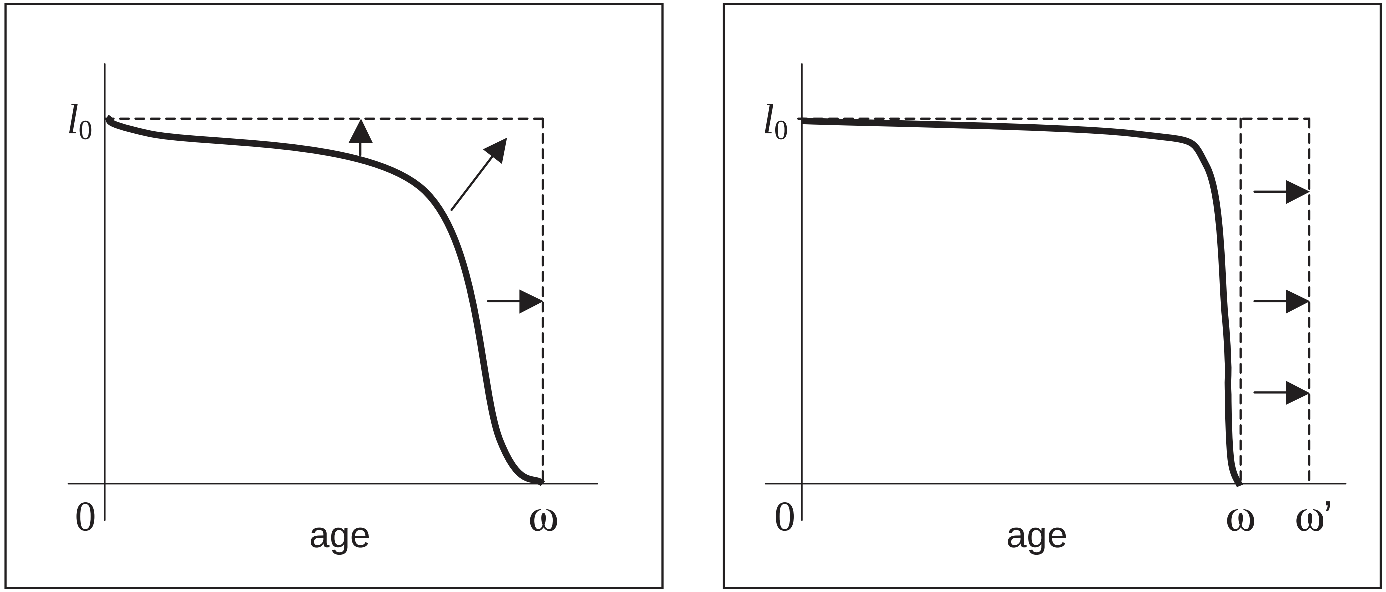
Survival functions. Source: ISTAT (males)

- rectangularization
- expansion

Uncertainty in future mortality trends *(cont'd)*



Curves of deaths. Source: ISTAT (males)

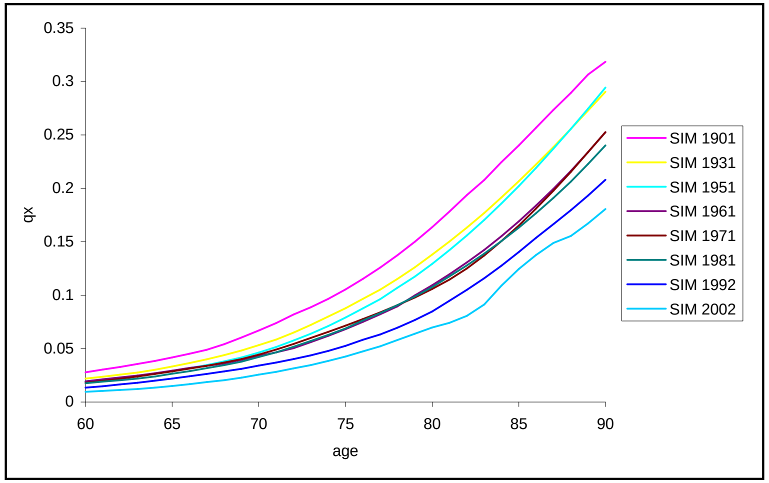


Rectangularization & Expansion

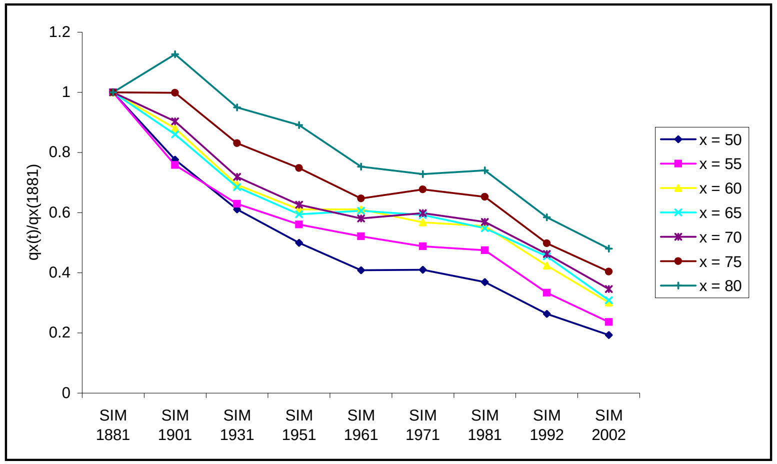
A (partial) shift from random fluctuations to systematic deviations

Risk of systematic deviations: *(aggregate) longevity risk*

Uncertainty in future mortality trends *(cont'd)*



(a)



(b)

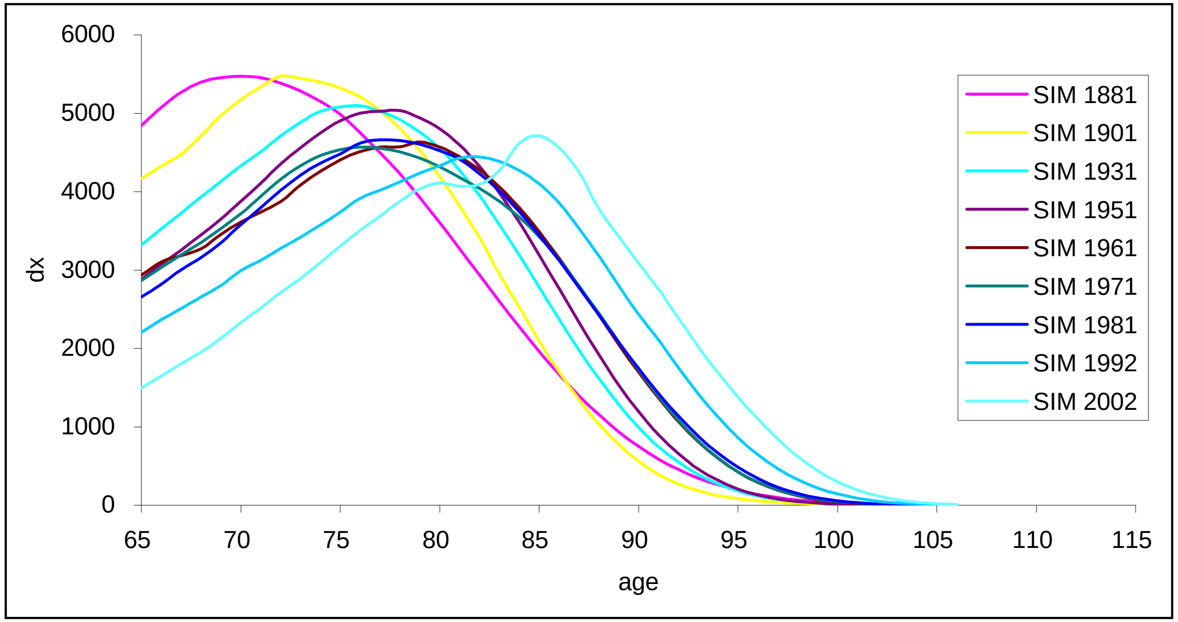
Mortality rates. Source: ISTAT (males)

Decreasing probabilities of death

See, for example:

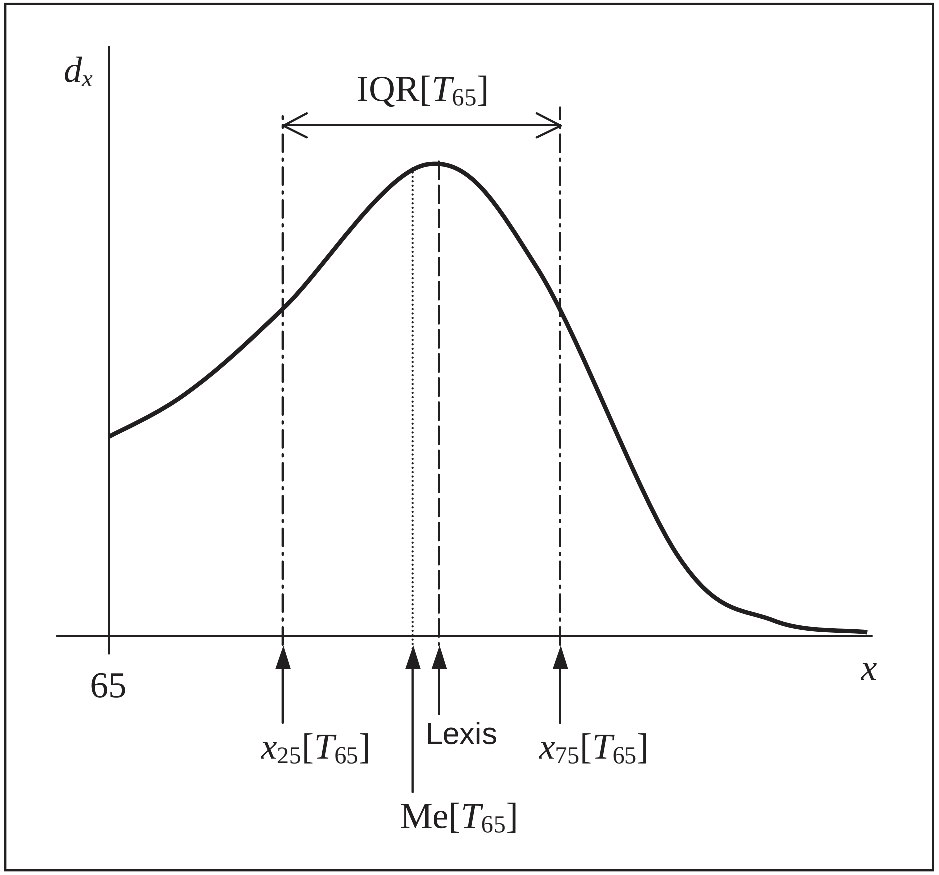
Tabeau et al. [2001], Willets et al. [2004], Pitacco [2004], Pitacco et al. [2009] and references therein

Uncertainty in future mortality trends *(cont'd)*



Curves of deaths referred to people alive at age 65. Source: ISTAT (males)

Rectangularization does not affect the probability distribution of the random lifetime, conditional on attaining, say, age 65



The curve of deaths referred to people alive at age 65. *Some markers*

	SIM 1881	SIM 1901	SIM 1931	SIM 1951	SIM 1961	SIM 1971	SIM 1981	SIM 1992	SIM 2002
$\mathrm{Me}[T_{65}]$	74.45827	75.09749	76.55215	77.42349	78.21735	77.94686	78.27527	80.23987	82.20066
$x_{25}[T_{65}]$	69.80944	70.45377	71.45070	72.16008	72.43802	72.32797	72.65518	73.89806	75.73235
$x_{75}[T_{65}]$	79.95515	80.14873	81.80892	82.63073	83.86049	83.84586	83.96275	86.02055	87.83705
$\mathrm{IQR}[T_{65}]$	10.14570	9.694965	10.35822	10.47065	11.42247	11.51789	11.30757	12.12249	12.10470

Probability distribution of the remaining lifetime at age 65*. Some markers*

Random fluctuations in lifetimes should not be underestimated when dealing with (small) life annuity portfolios and pension funds

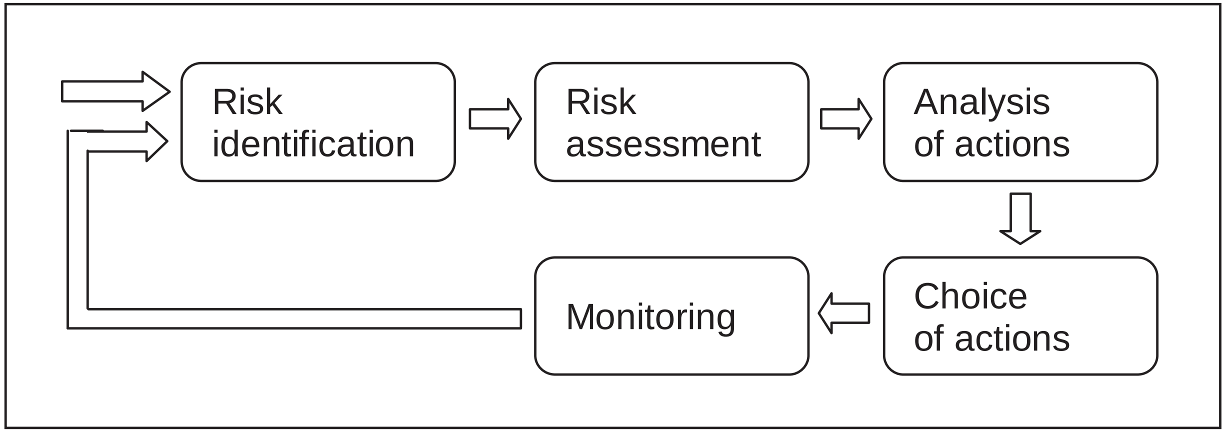
Projected life tables are needed, but the randomness (especially in future mortality trend) should also be allowed for

See, for example:

Olivieri [2001]

4 THE ERM FRAMEWORK

THE RISK MANAGEMENT PROCESS



Steps in the RM process

RISK IDENTIFICATION

See, for example:

International Actuarial Association [2004]

Basic issues

- Risk *sources* (or *causes*) (underwriting, market, operational, etc)
- Risk *components*, in particular:
  - ▷ process risk, i.e. the risk of random fluctuations
  - ▷ uncertainty risk, i.e. the risk of systematic deviations
- Risk *factors*, influencing the severity of impact on portfolio results (portfolio size, policy conditions, etc)

RISK ASSESSMENT

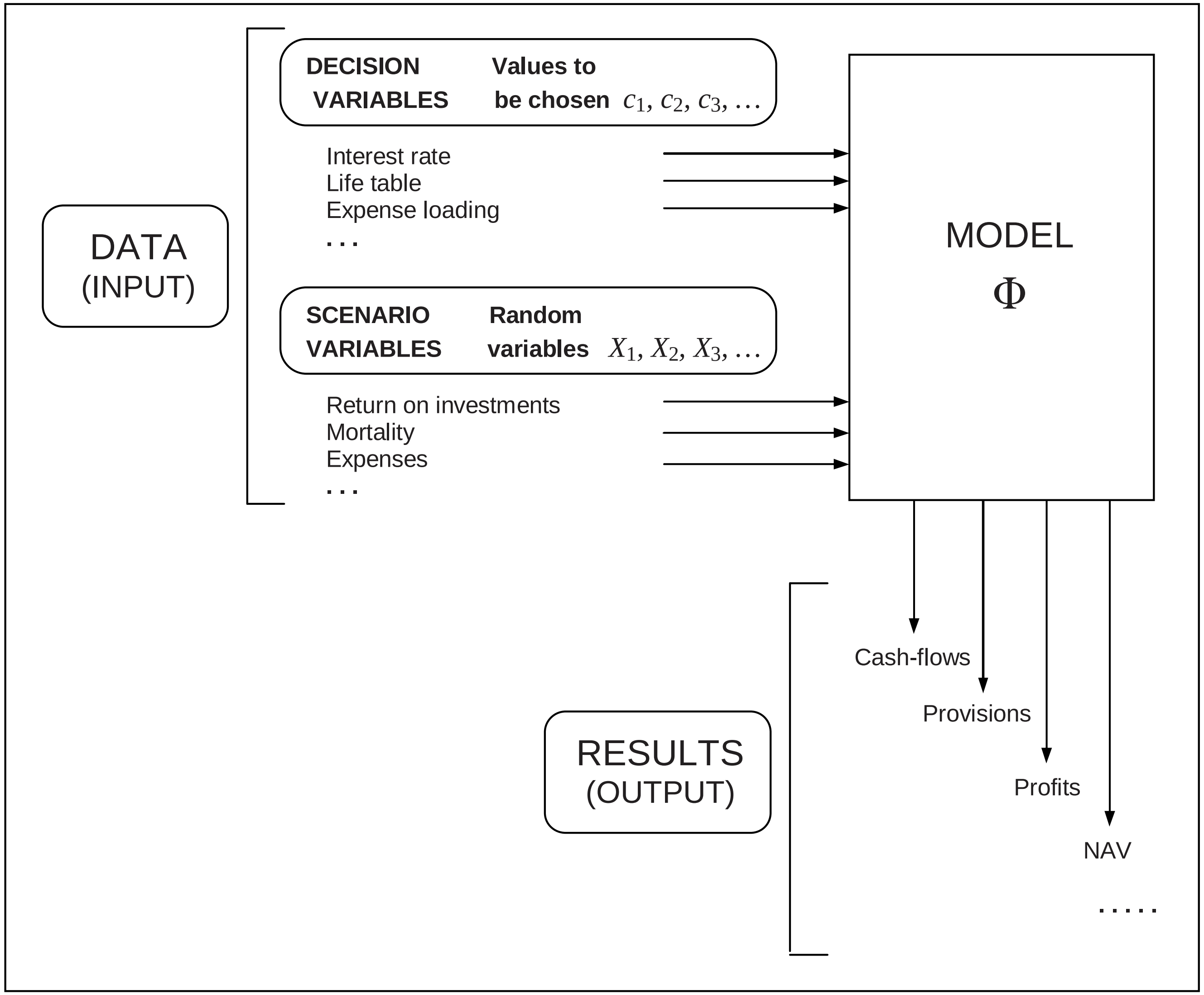
In general:

- $X_1, X_2, X_3, \ldots$: random variables representing (causes of) risks
- $c_1, c_2, c_3, \ldots$: values assigned to decision variables
- Y: a result chosen to assess the impact of risks

Then:

$$Y = \Phi(X_1, X_2, X_3, \ldots; c_1, c_2, c_3, \ldots)$$

See the following Figure



Modeling in life insurance: a comprehensive approach

How to implement the model ?

Ideal target: given

- ▷ the joint distribution of $(X_1, X_2, X_3, \dots)$

or

- ▷ the marginal distributions of $X_1, X_2, X_3, \dots$ and correlation assumptions (possibly via copula)

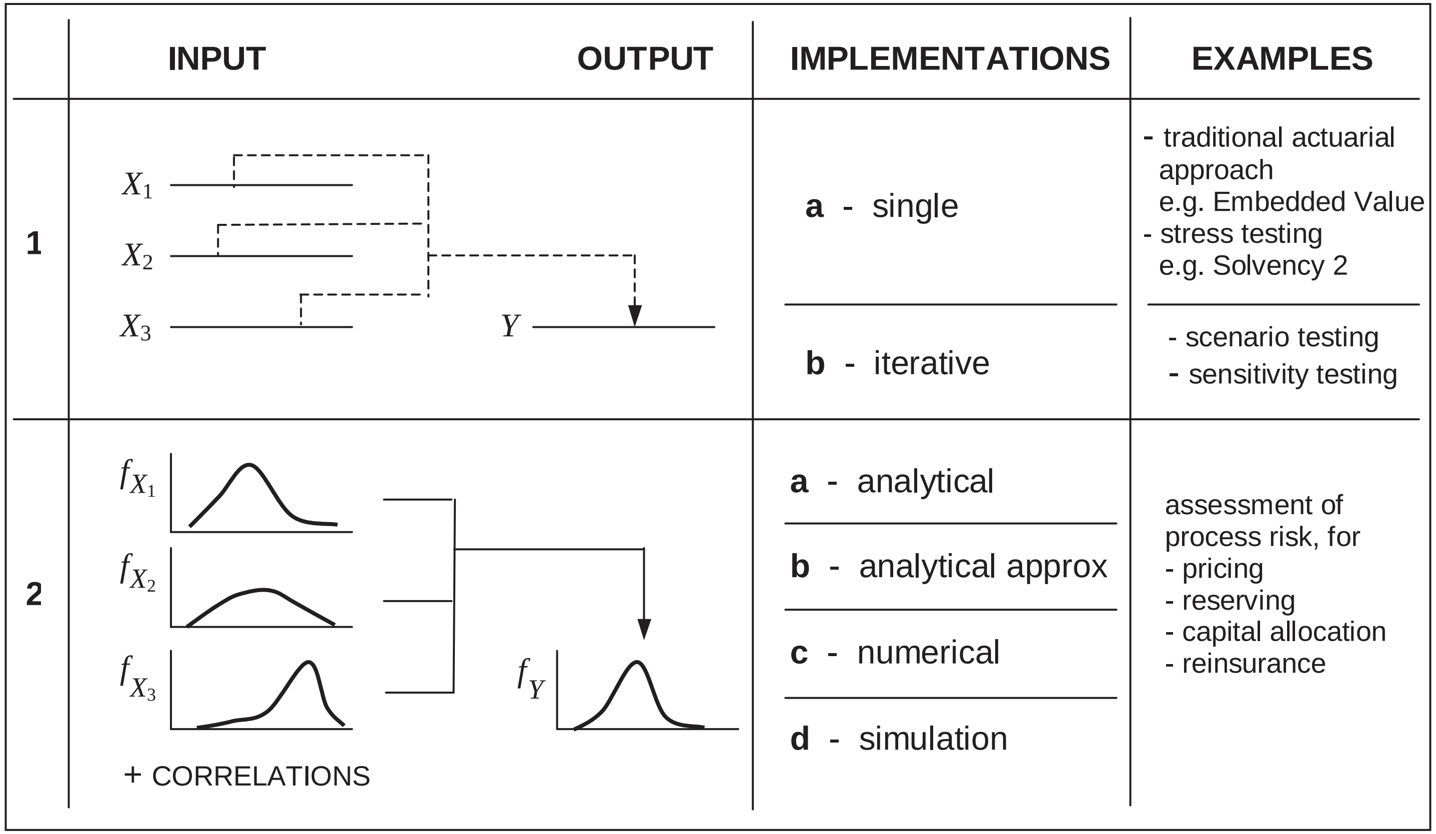
find the probability distribution of Y

In practice, (almost) impossible to find the probability distribution of Y via analytical procedures (heavy simplifications usually required)

A wide range of approaches available: from purely deterministic to "completely" stochastic

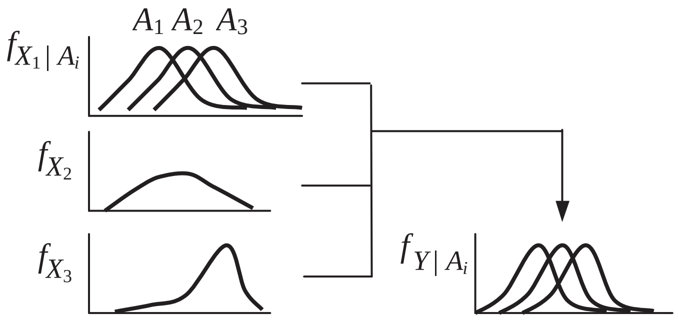
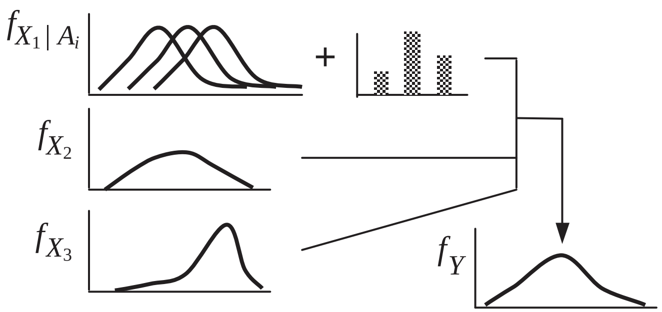
See the following Figures

The ERM framework *(cont'd)*



Implementing a stochastic model (1)

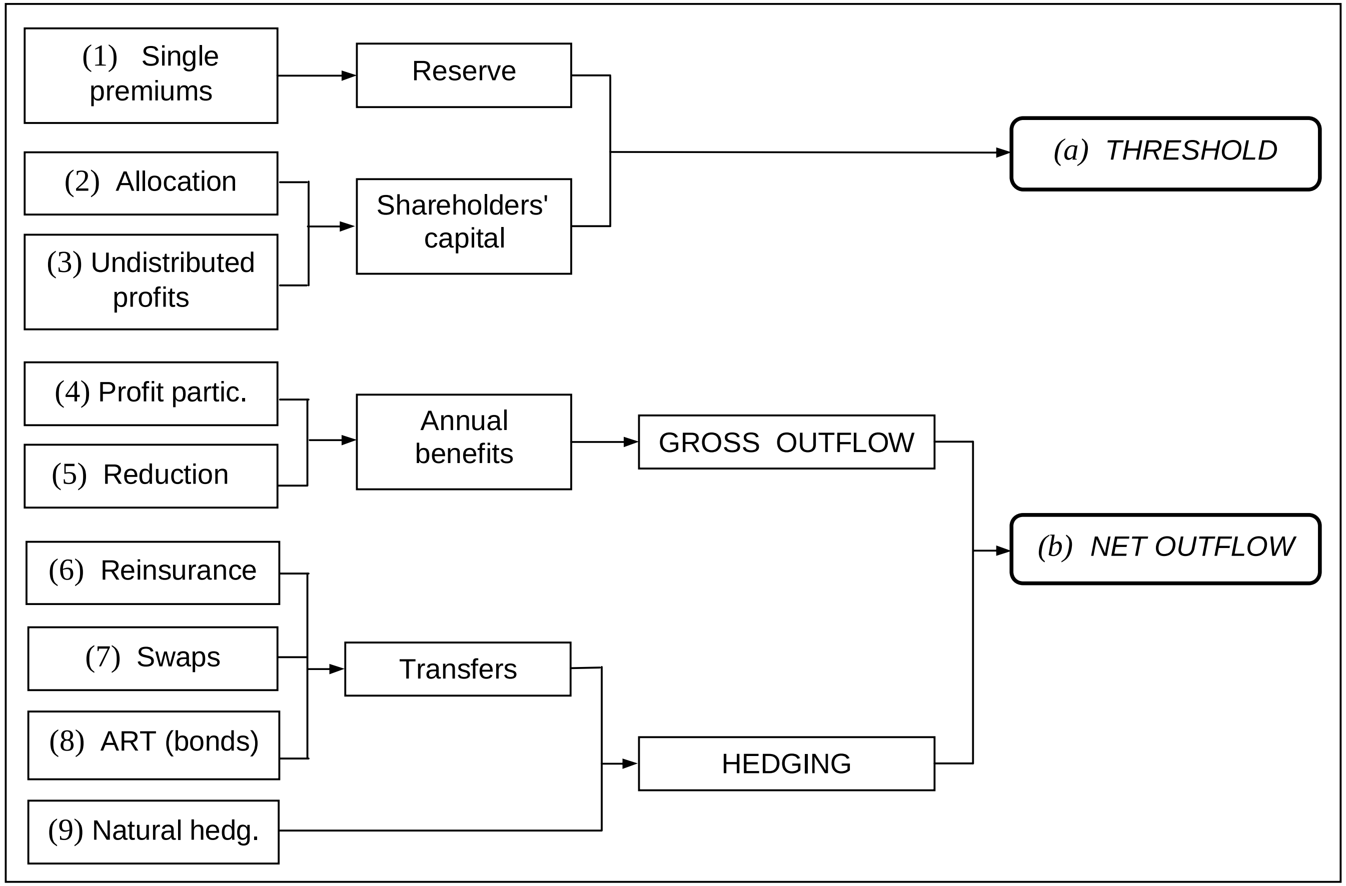
The ERM framework *(cont'd)*

	INPUT — OUTPUT	IMPLEMENTATIONS	EXAMPLES
3	 + CORRELATIONS	**a** - analytical **b** - analytical approx **c** - numerical **d** - simulation	assessment of process risk and scenario testing for uncertainty risk
4	 + CORRELATIONS	simulation	assessment of process risk and uncertainty risk

Implementing a stochastic model (2)

Analysis and choice of actions

Back to life annuity portfolio



Actions aiming at risk mitigation

Available actions (at least in principle)

$(1) \;\Rightarrow\; (a)$ - Pricing

- ▷ use of an appropriate projected table
- ▷ premium calculation principle

$(4) \;\Rightarrow\; (b)$ - Profit participation

$(5) \;\Rightarrow\; (b)$ - Reduction of the benefit (see Product design)

Remark

$(1), (4), (5)$: *loss control* actions

$(6) \;\Rightarrow\; (b)$ - Traditional reinsurance arrangements

- as regards the systematic component of longevity risk, traditional reinsurance does not provide by itself a solution, as the risk cannot be diversified by increasing (reinsurer's) portfolio size
- traditional reinsurance can work provided that:
  - ▷ the reinsurer experiences easier natural hedging (see below)
  - ▷ a further transfer (to capital markets) is feasible

$(7) \;\Rightarrow\; (b)$ - Swap-like reinsurance

$(8) \;\Rightarrow\; (b)$ - ART, viz longevity bonds

Remark

$(6), (7), (8)$: *risk transfer* actions

See, for example:

Blake and Burrows [2001], Blake et al. [2006], Olivieri [2005], Pitacco et al. [2009]

$(2) \Rightarrow (a)$, $(3) \Rightarrow (a)$ - Shareholders' capital

Remark

$(2), (3)$: *capital allocation* action

Capital allocation: a critical issue

- regulatory requirements
  - ▷ standard formula
  - ▷ (partial) internal models
- own assessment

Approaches:

- stochastic assessment
- deterministic requirements (e.g. -20% in probabilities of death)

See, for example:

Olivieri [2011], Olivieri and Pitacco [2003], Olivieri and Pitacco [2008], Olivieri and Pitacco [2009a], Olivieri and Pitacco [2009b]

$(9) \Rightarrow (b)$ - Natural hedging

- ▷ "across LOBs": insurance products with a negative sum at risk (viz life annuities) & insurance products with a positive sum at risk (endowments, assurances)
- ▷ "across time": life annuities with death benefit (capital protection) which decreases as the age at death increases

What about the effectiveness ?

Remark

(9): *internal risk reduction*

MONITORING

Objectives of the monitoring phase:

- ▷ checking the effectiveness of the undertaken actions
- ▷ determining whether changes in the scenario suggest novel solutions

Sound monitoring requires appropriate modelling structures

Two examples of monitoring-oriented modelling

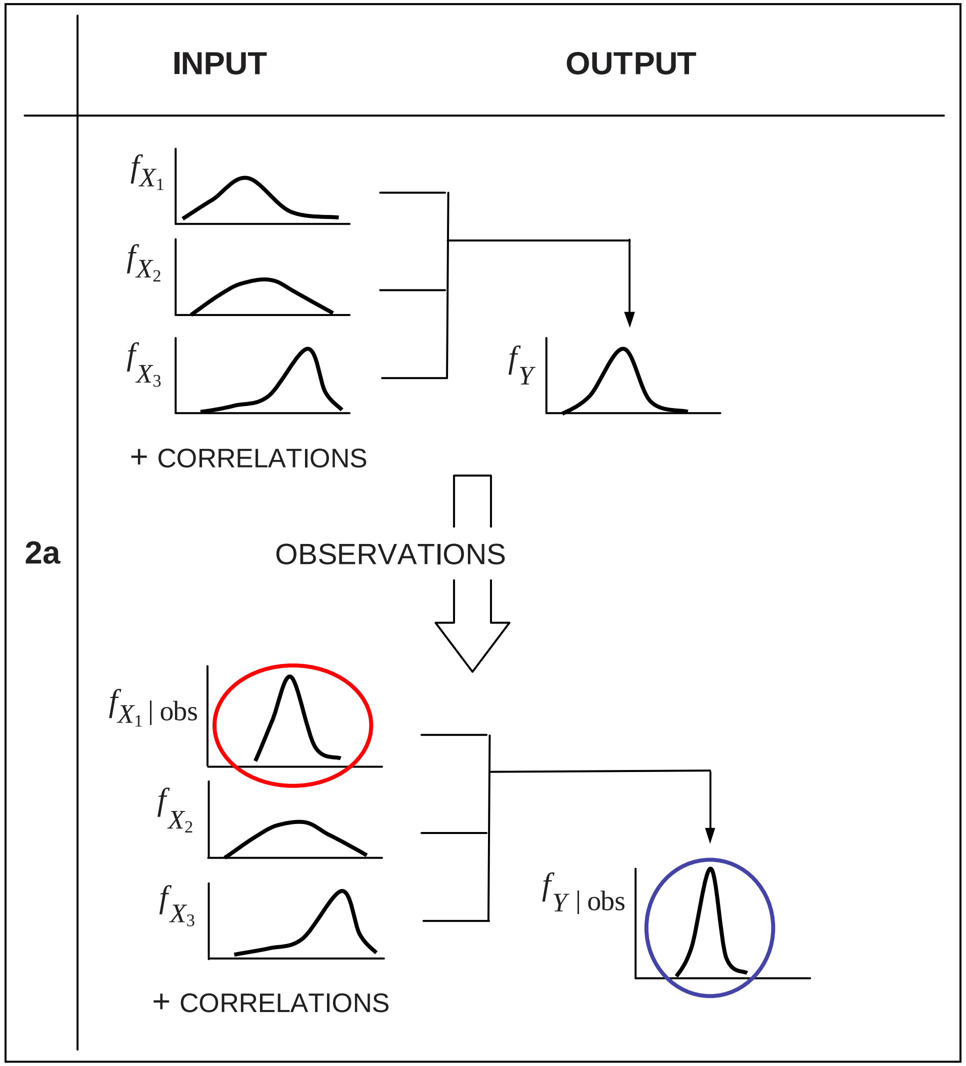
See:
Olivieri and Pitacco [2009a], Olivieri [2011], Olivieri and Pitacco [2012]

(1) $\Rightarrow$ updating the probability distributions of the numbers of death

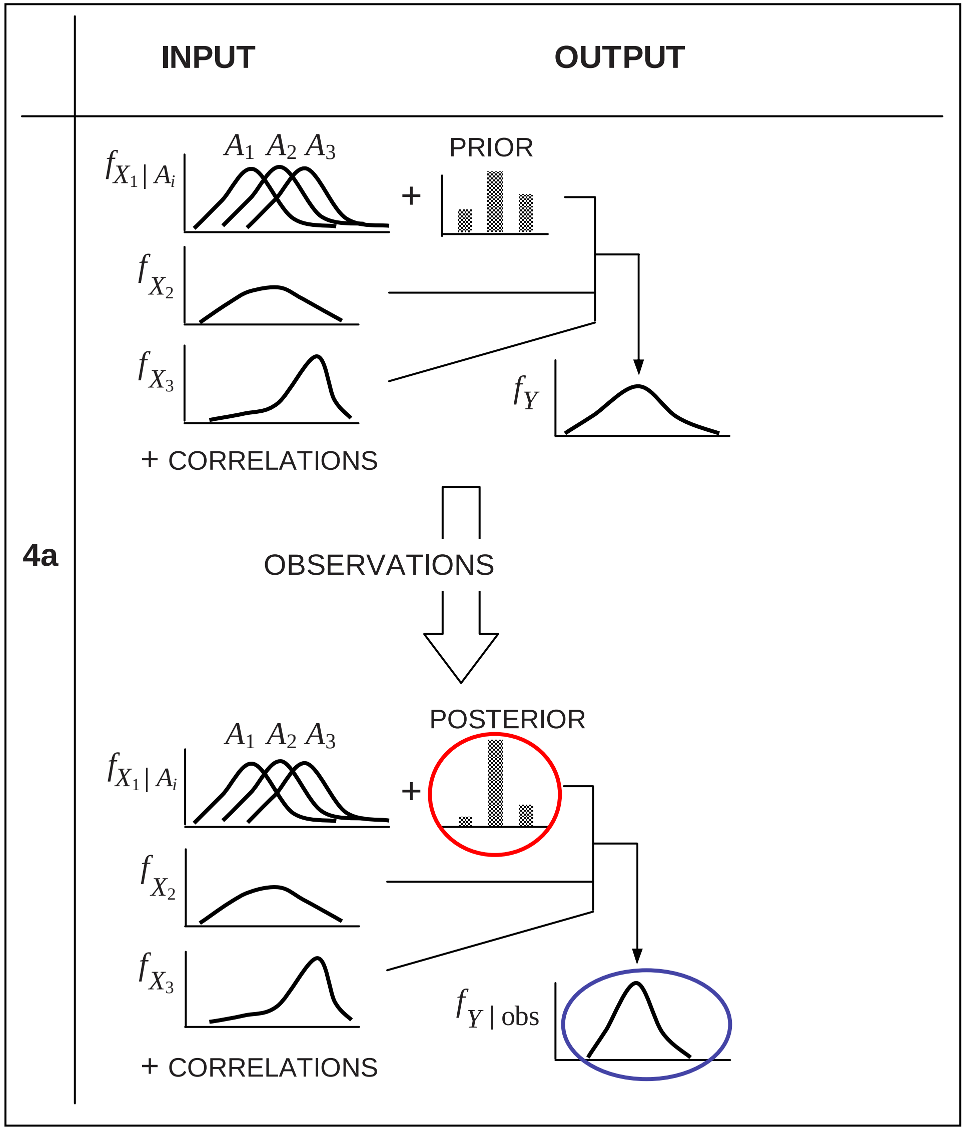
See:
Olivieri and Pitacco [2002]

(2) $\Rightarrow$ updating the probability distribution on the scenario space (each scenario representing an assumption about future mortality trend)

The ERM framework *(cont'd)*



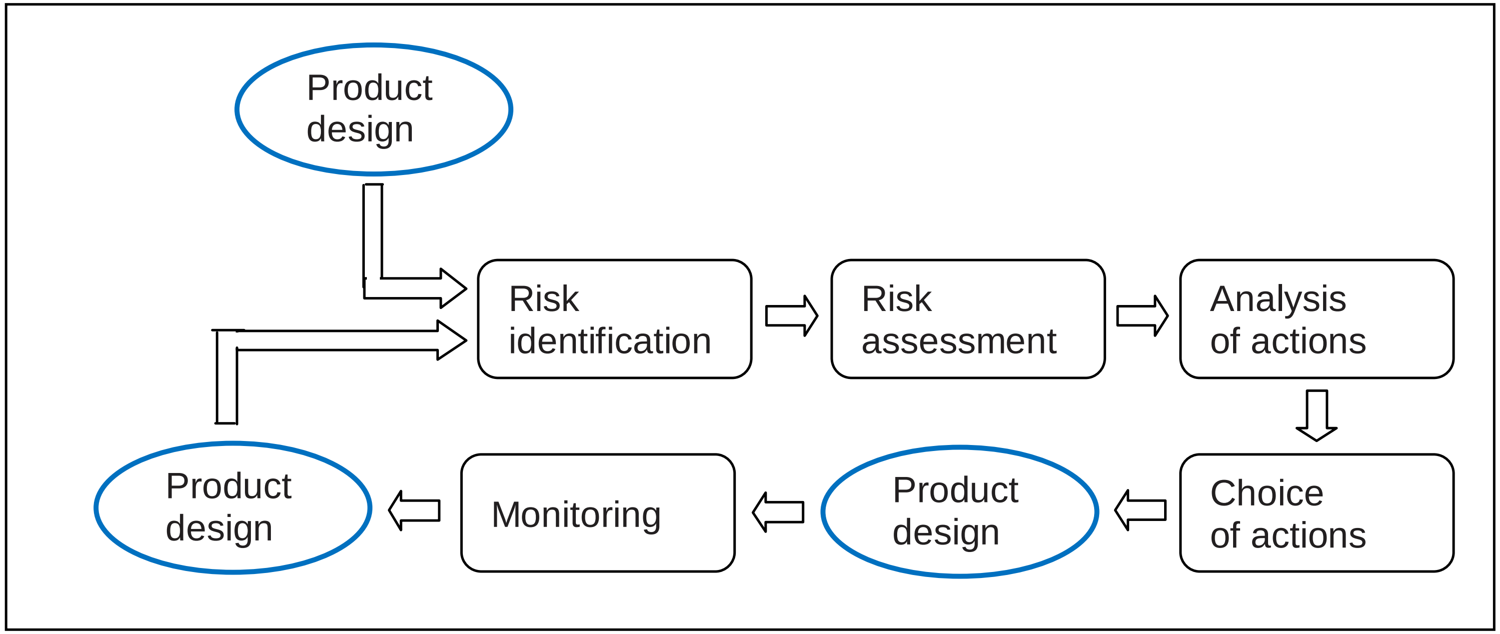
A stochastic model allowing for experience (1)



A stochastic model allowing for experience (2)

Rethinking the Risk Management process

Product design: a RM action



Steps in the RM process: an extension

Product design aiming at improving the annuity provider's risk profile, via:

(1) weakening the interest guarantee

(2) weakening the longevity guarantee

As regards (1): in life annuities with participation in investment results, possible shift from annual interest rate guarantee (implying annual lock-in, i.e. a cliquet-like option) to multi-year average interest rate guarantee (with periodic lock-in, say every 5 years)

We focus on (2)

5 PRODUCT DESIGN: SHARING THE LONGEVITY RISK BETWEEN ANNUITANTS AND ANNUITY PROVIDER

Conventional life annuity:

- ▷ deterministic benefit, e.g. flat profile b
- ▷ benefit payment also relies on “mortality credits”, i.e. release of reserves pertaining to died annuitants ($\Rightarrow$ *mutuality*)
- ▷ longevity risk originated by possible number of deaths lower than expected, borne by the annuity provider

Sharing the longevity risk $\Rightarrow$ linking the annual benefit to some measure of mortality

First we describe some specific solutions adopted in insurance and pension practice

See, for example:

Pitacco et al. [2009]

Then, we refer to a more general framework and a more systematic approach to the problem of sharing the risk between annuitants and annuity provider

SHARING THE (FUTURE) RISK DURING THE ACCUMULATION PHASE

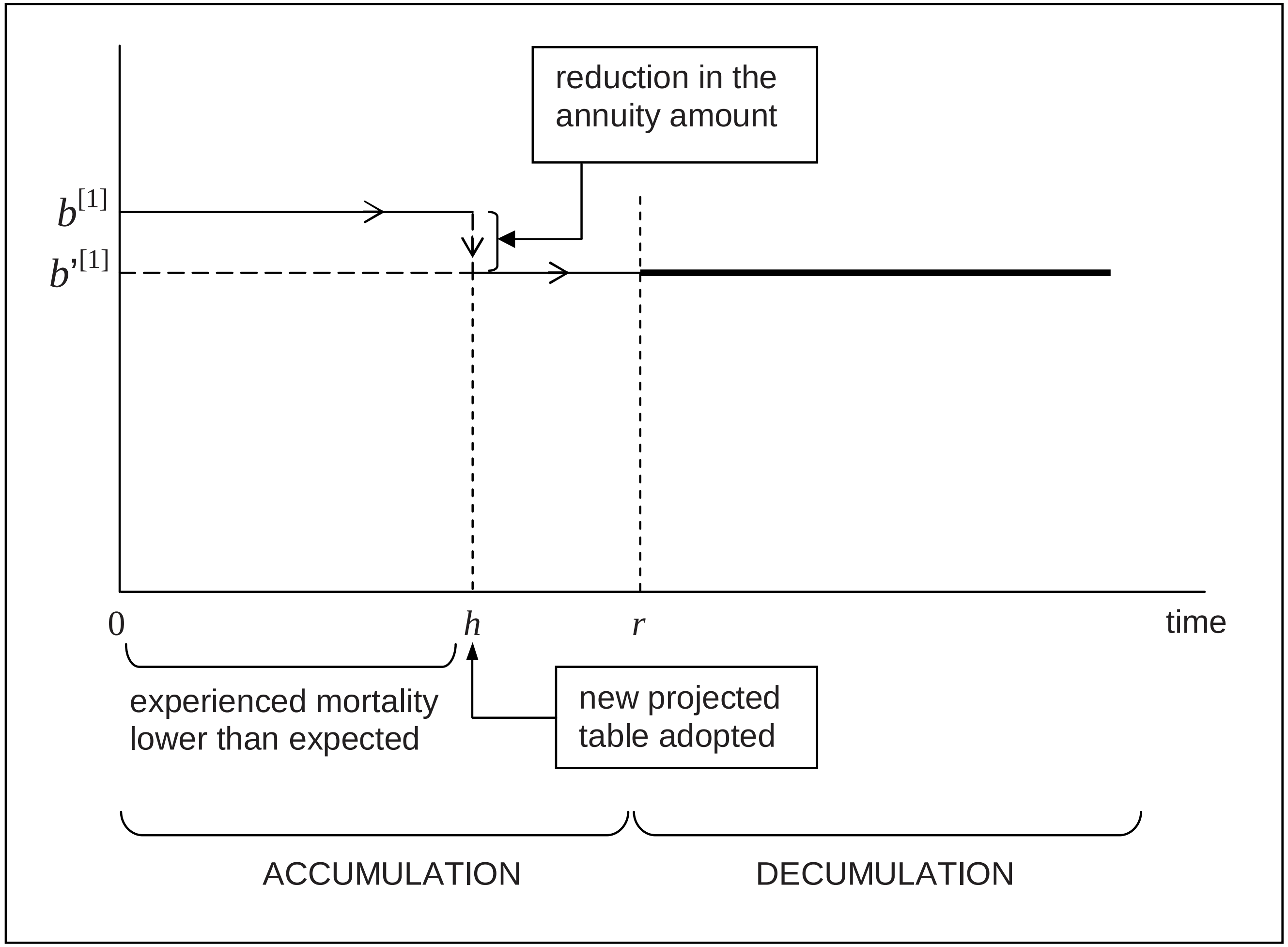
Rigorous approach to life annuity assessment
$\Rightarrow$ high premium rates and / or appropriate shareholders' capital allocation

Alternative:

- ▷ lower premium rates and / or less capital
- ▷ in the case of mortality improvement much higher than expected $\Rightarrow$ new projected life table adopted $\Rightarrow$ reduction of the (future) benefit

$$b^{[1]} \longrightarrow b'^{[1]}$$

Product design: sharing the longevity risk . . . *(cont'd)*



Sharing the (future) longevity risk during the accumulation phase

Constraints in the arrangement (e.g. imposed by the supervisory authority)

- ▷ mortality improvement must be beyond a fixed threshold (for example, in terms of a raise in life expectancy at 65)
- ▷ benefit reduction applied at the latest a stated time before the end of the accumulation phase (say, 2 years)
- ▷ no more than 1 reduction every k years (e.g. $k = 5$)
- ▷ whatever the mortality improvements, the total reduction cannot be greater than a stated percentage ρ (e.g. 15%); thus (assuming just one reduction)

$$\frac{b^{[1]} - b'^{[1]}}{b^{[1]}} \leq \rho$$

$\Rightarrow$ guaranteed benefit = $(1 - \rho)\, b^{[1]}$

SHARING THE RISK IN THE DECUMULATION PHASE

Assume that premium rates are kept high; then for a given single premium we obtain

$$b^{[2]} < b^{[1]}$$

In case of mortality improvements lower than expected, mortality profits arise

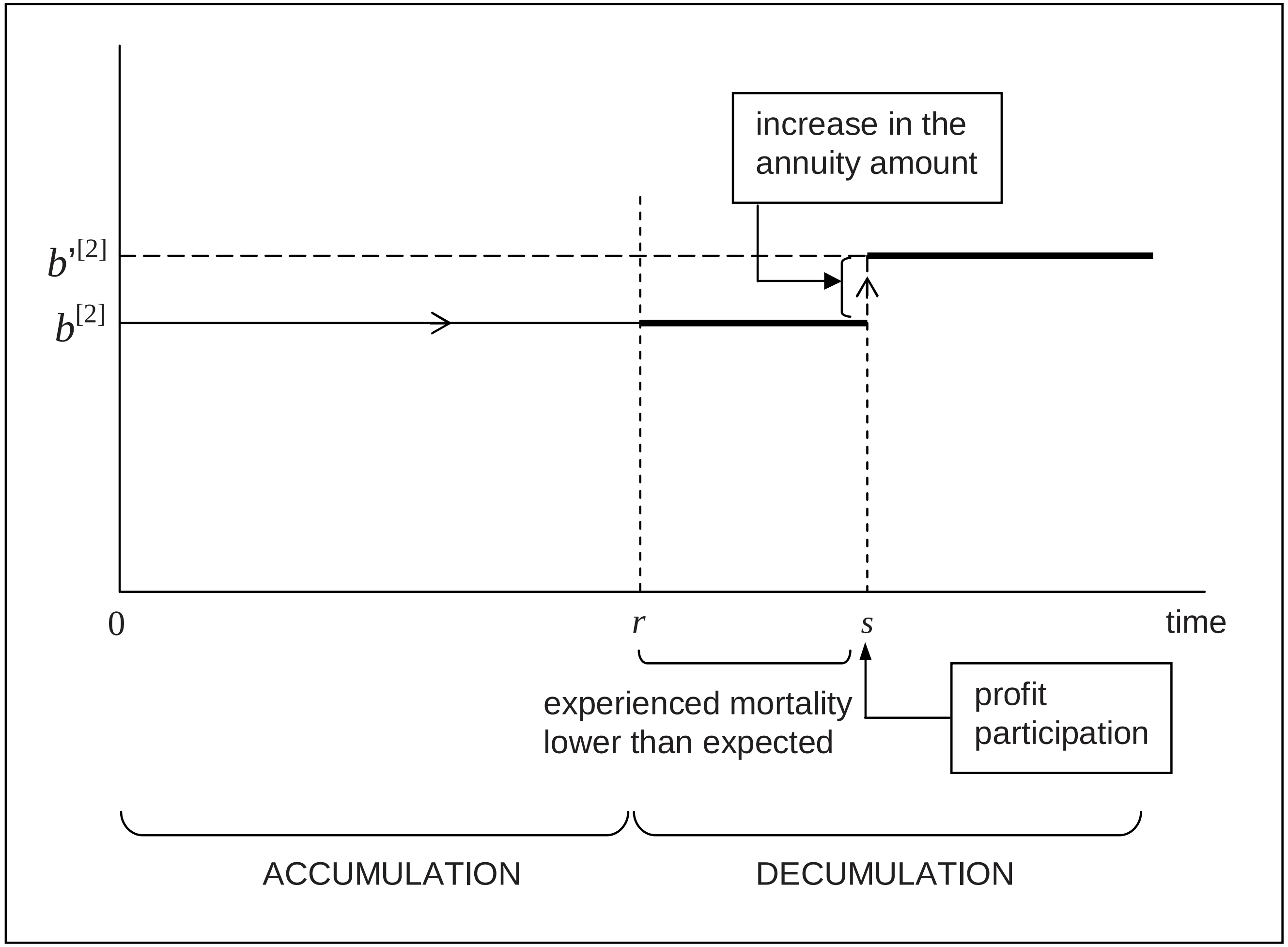
Profits can be distributed $\Rightarrow$ raise in the annual benefit

$$b^{[2]} \longrightarrow b'^{[2]}$$

$\Rightarrow$ participation in mortality profits

Problem: is this a tontine scheme ?

Product design: sharing the longevity risk . . . *(cont'd)*



Sharing the longevity risk in the decumulation phase

Assume that, conversely, premium rates are kept high, so that a benefit $b^{[2]}$ should be paid

If premium rates are considered very high in relation to likely mortality trend, a benefit $b^{[3]}$ is initially stated (more advantageous for policyholders)

In case of mortality improvements higher than expected $\Rightarrow$ reduction of the benefit even during the decumulation period:

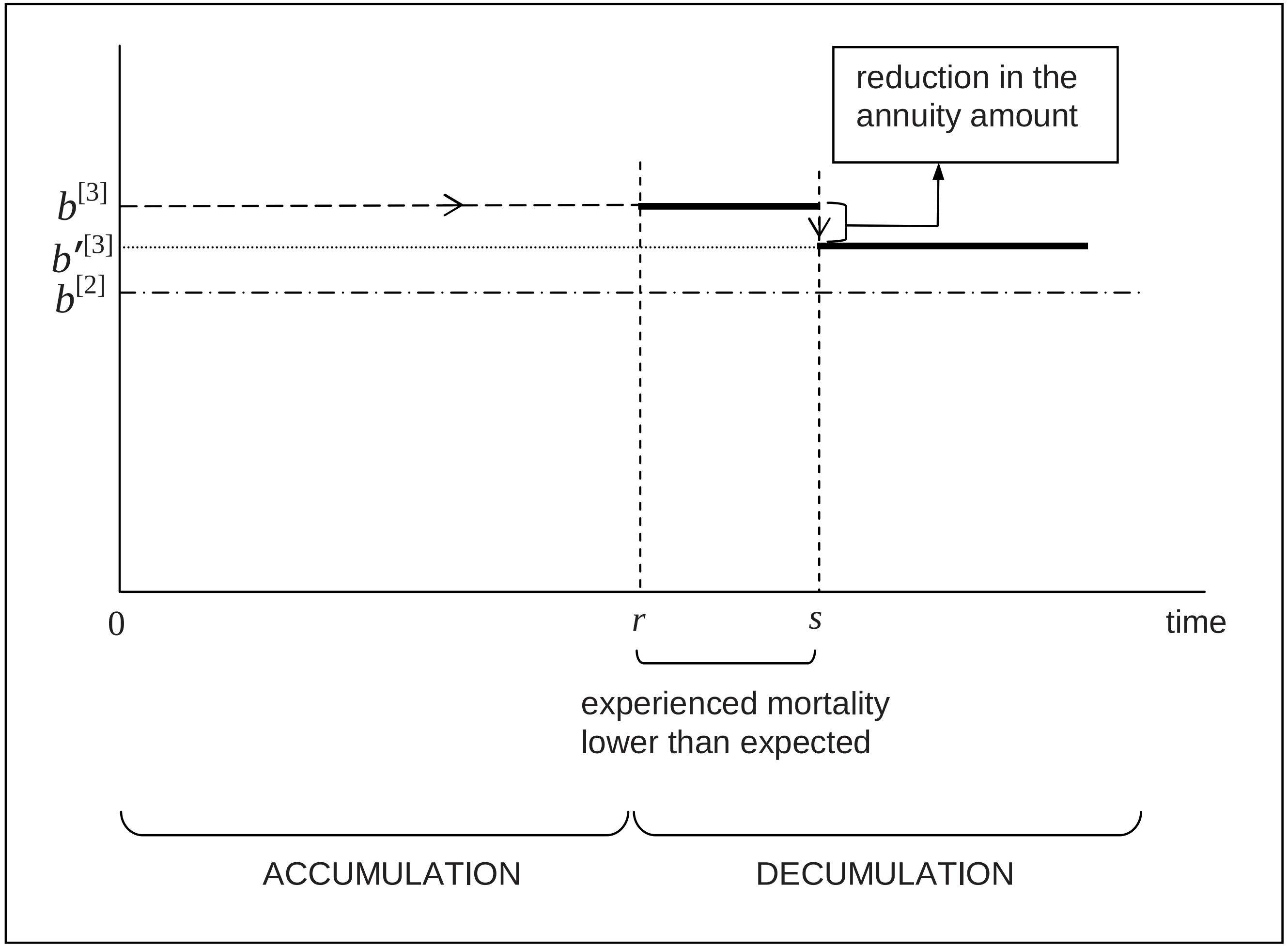
$$b^{[3]} \longrightarrow b'^{[3]}$$

with

$$b'^{[3]} \geq b^{[2]}$$

Note: the guaranteed benefit is $b^{[2]}$ (not $b^{[3]}$)

Product design: sharing the longevity risk . . . *(cont'd)*



Sharing the longevity risk in the decumulation phase

DECUMULATION PHASE: A MORE SYSTEMATIC APPROACH

Previous examples: benefit b as a function of some measure of mortality trend

In more general terms $\Rightarrow$ *Adjustment process* $\Rightarrow$ benefit b_t due at time t:

$$b_t = b_0\, \alpha_t^{[\mathrm{m}]}$$

with $\alpha_t^{[\mathrm{m}]}$ = coefficient of adjustment over $(0, t)$, according to mortality trend measure $[\mathrm{m}]$

At annuity inception: random behavior of mortality $\Rightarrow$ random annual benefit B_t, at time t

Various interesting contributions regarding practicable models for the adjustment process

See:

Denuit et al. [2011], Goldsticker [2007], Kartashov et al. [1996], Lüty et al. [2001], Olivieri [2013], Piggott et al. [2005], Richter and Weber [2011], Sherris and Qiao [2011], van de Ven and Weale [2008]

Basic problems in defining the adjustment process:

1. choice of the age pattern of mortality referred to
2. choice of the link between annual benefits and mortality

Reasonable aim: sharing the *aggregate* longevity risk (i.e. the systematic component of the longevity risk), leaving the volatility (i.e. the random fluctuation component) with the annuity provider

1. Examples of *mortality referred to*
   (a) Actual number of surviving annuitants

$$n_{x+1},\ n_{x+2}, \dots$$

   (b) Actual number of survivors in the "reference" cohort

$$l_{x+1},\ l_{x+2}, \dots$$

   (c) Expected number of surviving annuitants, according to (initial) information $\mathcal{F}$ (for example: $\mathcal{F}$ = life table)

$$\mathbb{E}[N_{x+1} \,|\, \mathcal{F}],\ \mathbb{E}[N_{x+2} \,|\, \mathcal{F}], \dots$$

(d) Expected number of survivors in the reference cohort, according to (initial) information $\mathcal{F}$

$$\mathbb{E}[L_{x+1} \,|\, \mathcal{F}], \mathbb{E}[L_{x+2} \,|\, \mathcal{F}], \dots$$

(e) Expected number of surviving annuitants, according to (current) updated information $\mathcal{F}'$

$$\mathbb{E}[N_{x+t} \,|\, \mathcal{F}'], \mathbb{E}[N_{x+t+1} \,|\, \mathcal{F}'], \dots$$

for example: $\mathcal{F}' = \{\mathcal{F}; n_{x+1}, \dots, n_{x+t-1}\}$;

See:

Olivieri and Pitacco [2009a], Olivieri and Pitacco [2012]

(f) Expected number of survivors in the reference cohort, according to (current) updated information $\mathcal{F}^*$

$$\mathbb{E}[L_{x+t} \,|\, \mathcal{F}^*], \mathbb{E}[L_{x+t+1} \,|\, \mathcal{F}^*], \dots$$

for example: $\mathcal{F}^*$ = new projected life table

Product design: sharing the longevity risk . . . *(cont'd)*

Reference cohort: a cohort in a population, which should have

- ▷ age-pattern of mortality
- ▷ mortality trend

close to those in the portfolio or pension fund

Reference cohort should be referred to (instead of annuitants in the portfolio or pension plan) for objectivity and transparency reasons

However, *basis risk* arises when linking adjustments to a reference cohort, because of possible mortality trend different from the one experienced in the portfolio or pension fund

2. Definition of the *adjustment coefficients*

   Various approaches can be adopted

   In particular the definition can be

   ▷ *retrospective*: directly involving observed mortality, in terms of
   $n_{x+1},\ n_{x+2}, \dots$
   or
   $l_{x+1},\ l_{x+1}, \dots$

   ▷ *prospective*: relying on updated mortality forecasts, e.g.
   $\mathbb{E}[L_{x+t} \,|\, \mathcal{F}^*],\ \mathbb{E}[L_{x+t+1} \,|\, \mathcal{F}^*], \dots$

   Quantities involved:

   - $\ddot{a}_{x+t}^{[\mathcal{F}]}$ = actuarial value of an annuity, according to information $\mathcal{F}$
   - $V_t^{[\mathcal{F}]}$ = individual reserve at time t
   - $V_t^{[\mathrm{P},\mathcal{F}]}$ = portfolio reserve at time t, according to information $\mathcal{F}$
   - A_t = assets available at time t

(a) Example 1 of the retrospective approach. Define:

$$\alpha_t^{[1]} = \frac{\mathbb{E}[L_{x+t} \,|\, \mathcal{F}]}{\mathbb{E}[L_x \,|\, \mathcal{F}]} \, \frac{n_x}{n_{x+t}}$$

Result: $V_{t^+}^{[\mathrm{P},\mathcal{F}]}$ expected value at time 0 of the portfolio reserve

(b) Example 2 of the retrospective approach. Define:

$$\alpha_t^{[2]} = \frac{A_t}{V_t^{[\mathcal{F}]}}$$

Result: $V_{t^+}^{[\mathrm{P},\mathcal{F}]} = A_t$ = available assets

Note that:

- ▷ both volatility and aggregate longevity risk borne by the annuitants
- ▷ market risk also borne by the annuitants
- ▷ arrangement characterizing (pure) Group Self-Annutization (GSA)

(c) Example of the prospective approach. Define:

$$\alpha_t^{[3]} = \frac{\ddot{a}_{x+t}^{[\mathcal{F}]}}{\ddot{a}_{x+t}^{[\mathcal{F}^*]}}$$

Result: $b_t \, \ddot{a}_{x+t}^{[\mathcal{F}^*]} = b_0 \, \ddot{a}_{x+t}^{[\mathcal{F}]}$

and hence: $V_{t^+}^{[\mathrm{P},\mathcal{F}^*]} = V_t^{[\mathrm{P},\mathcal{F}]}$

Some numerical results

- One cohort, all individuals initial age $x = 65$
- Mortality/longevity adjustments every $k = 5$ years
- Maximum age for mortality/longevity adjustment (apart from the GSA, i.e. $\alpha_t^{[2]}$): 95 (i.e., time 30)
- $\dfrac{A_{\omega-x}}{A_0}$: remaining assets at cohort's exhaustion, as a percentage of the initial assets (initial assets are funded just through premiums)
- Traditional premium calculation (equivalence principle):

$$A_0 = n_x \times \mathbb{E}[a_{\overline{K_x}\rceil}|\mathcal{F}]$$

Experienced mortality: 90% of the best-estimate (as at time 0, i.e. $\mathcal{F}$)
No extra-return on investments
New projected life table at time 10, yielding a higher life expectancy

t	no adj	$\alpha_t^{[1]}$	$\alpha_t^{[2]}$	$\alpha_t^{[3]}$
0	1.000	1.000	1.000	1.000
5	1.000	0.996	0.996	1.000
10	1.000	0.993	0.872	0.880
15	1.000	1.007	1.031	1.000
20	1.000	1.007	1.054	1.000
25	1.000	1.000	1.105	1.000
30	1.000	0.997	1.243	1.000
35	1.000	1.000	1.684	1.000
40	1.000	1.000	3.372	1.000
$\frac{b_{95-x}}{b_0}$	100.00%	98.03%	129.70%	87.98%
$\frac{A_{\omega-x}}{A_0}$	-8.554%	-7.580%	0.180%	9.467%

6 CONCLUDING REMARKS

Traditional actuarial mathematics and technique mainly rely on the calculation of expected values (viz in pricing and reserving) of *benefits* (sum in case of death, lifelong annuity benefits, etc.)

An appropriate stochastic approach is however required because of

- ▷ awareness of the presence of guarantees implying risks
- ▷ evolving scenarios
- ▷ the need for a sound assessment of the insurer's risk profile

As regards in particular mortality / longevity risk, a rigorous stochastic approach should be adopted

However

- implementation of complex stochastic models may constitute an obstacle on the way towards sound pricing
- facing the risks by charging very high premiums can reduce the insurer's market share

Alternative solution: appropriate product designs which aim at sharing risks between annuity provider and annuitants, or between insurer and policyholders

Weakening guarantees and simplifying the products do not exempt insurers and annuity providers from a sound (but hopefully simpler) assessment of the risk profile of portfolios and pension funds

Life annuities: severe solvency requirements (see Solvency 2) because of the aggregate longevity risk

See, for example:

Olivieri [2011], Olivieri and Pitacco [2009a], Olivieri and Pitacco [2009b]

Sharing the longevity risk $\Rightarrow$ less "absorbing" annuity and pension products (in particular as regards solvency regulation)

Main problems

- to find appropriate "reference" longevity
- to link effectively benefits to reference longevity

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